A mathematical consideration on the effect of regional lockdown on the final epidemic size

Zhiqiong Fu* and Hiromi Seno

Department of Computer and Mathematical Sciences
Graduate School of Information Sciences
Tohoku University, Sendai, Japan

*fu.zhiqiong.t6@dc.tohoku.ac.jp

Regional lockdown can be considered as a policy to suppress the spread of a transmissible disease when the epidemic breaks out. The government could adopt different levels of lockdown policies according to the epidemic situation. In this work, we propose a mathematical model to discuss the effect of different levels of lockdown on the final epidemic size in a community. Assumptions for our modeling are given as follows:

- The community is composed of the regional area (area 1) and the central area (area 2) with different qualities about the medical treatment for a transmissible disease;
- The disease is not fatal;
- Susceptible individuals can temporarily visit the other area;
- Some infective individuals of the regional area can get the medical treatment at the central area, for example, transported by ambulance;
- Recovered individuals become susceptible again;
- Population size is constant in each area according to the epidemic dynamics.

We indicate the individuals of the area i who are healthy and can be infected by S_i; those of the area i who have been infected and are able to transmit the disease by I_i; the infectives of the area j who are getting the medical treatment at the area i by H_{ij}. Then, our mathematical model is expressed by the following system of ordinary differential equations:

$$\begin{aligned}
\frac{dS_1}{dt} &= -\beta_1 I_1 S_1 - \alpha_1 \beta_2 I_2 S_1 + \theta_1 H_{11} + \theta_2 H_{21} \\
\frac{dI_1}{dt} &= \beta_1 I_1 S_1 + \alpha_1 \beta_2 I_2 S_1 - \gamma_{11} I_1 - \gamma_{21} I_1 \\
\frac{dH_{11}}{dt} &= \gamma_{11} I_1 - \theta_1 H_{11} \\
\frac{dH_{21}}{dt} &= \gamma_{21} I_1 - \theta_2 H_{21} \\
\frac{dS_2}{dt} &= -\beta_2 I_2 S_2 - \alpha_2 \beta_1 I_1 S_2 + \theta_2 H_{22} \\
\frac{dI_2}{dt} &= \beta_2 I_2 S_2 + \alpha_2 \beta_1 I_1 S_2 - \gamma_{22} I_2 \\
\frac{dH_{22}}{dt} &= \gamma_{22} I_2 - \theta_2 H_{22},
\end{aligned}$$

where β_i is the infection coefficient in the area i; $\alpha_i \beta_j$ is the infection coefficient during the temporary visit to the area j, which is smaller than β_j $(0 < \alpha_i < 1)$; γ_{ij} is the treatment rate that the infective of the area j gets the medical treatment at the area i; θ_i is the recovery rate by the medical treatment at the area i.

In this work, based on the above mathematical model, we consider different levels of restrictions on individuals' movement as the condition to distinguish the types of lockdown policy: weak lockdown, strong lockdown, and complete lockdown. In order to discuss the effects of lockdown policies at different levels, we will analyze and compare the final epidemic size for each type of lockdown policy.