An SIRI+Q Model with Limited Capacity of Isolation

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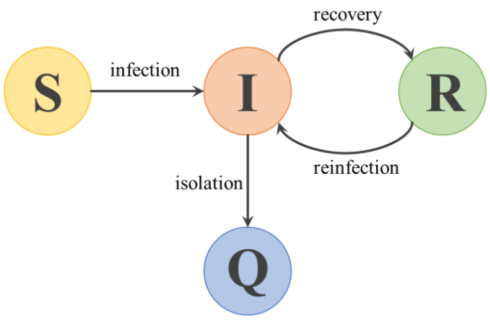
Isolation is one of the effective methods to suppress the spread of an epidemic in the community. In this work, we consider the following mathematical model on the epidemic dynamics with a reinfectious disease, containing the isolation class Q. The isolation is limited by a given finite capacity. We shall focus on the relation of the isolation capacity to the epidemic consequence.



$$\frac{dS}{dt} = -\beta \frac{I}{N-Q} S$$

$$\frac{dI}{dt} = \beta \frac{I}{N-Q} S + \varepsilon\beta \frac{I}{N-Q} R - \rho I - \Phi(Q, I)$$

$$\frac{dQ}{dt} = \Phi(Q, I)$$

$$\frac{dR}{dt} = \rho I - \varepsilon\beta \frac{I}{N-Q} R$$

with

$$\Phi(Q, I) = \begin{cases} \sigma I & \text{for } Q < Q_{\max} \\ 0 & \text{for } Q = Q_{\max} \end{cases}$$

and the initial condition $(S(0), I(0), Q(0), R(0)) = (S_0, I_0, 0, 0)$ where $I_0 > 0$ and $S_0 = N - I_0 > 0$. Parameter β is the infection coefficient, $\varepsilon\beta$ is the reinfection coefficient $(0 < \varepsilon < 1)$, ρ is the natural recovery rate of the infective individual. The piece-wise function $\Phi(Q, I)$ denotes the isolation rate of the infected individual. As long as the isolated subpopulation size Q is less than the isolation capacity $Q_{\max}$, the isolation is available, and the epidemic dynamics remains at the isolation effective phase with $\Phi(Q, I) = \sigma I$. However, the isolation ceases once Q reaches $Q_{\max}$. Then the epidemic dynamics transfers to the isolation incapable phase with $\Phi(Q, I) = 0$. Any demographic change due to birth, death or migration is assumed to be negligible in the epidemic season. The isolated individual cannot contact others or be discharged in the epidemic season.

In this presentation, we are going to show some essential mathematical results on the importance of sufficient isolation capacity for the control of epidemics. Our results imply that a sufficient isolation capacity could effectively reduce both the final epidemic and endemic sizes for a reinfectious disease, while the insufficient capacity could cause an unexpectedly large sizes of them.