A discrete-time model for the annual sequence of cumulative incidences by recurring SIR epidemic dynamics with waning immunity

免疫失活を伴う反復 SIR 感染症伝染ダイナミクスによる累積罹患率の年次変動の離散時間モデル

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Today, a variety of commonly recognized seasonal infectious diseases cause their spread during specific epidemic seasons: common cold（普通感冒）, influenza（流行性感冒）, respiratory syncytial virus (RSV)（RS ウイルス感染症）, viral gastroneteritis（ウイルス性胃腸炎）, malaria, dengue, chikungunya, cholera, COVID-19, etc. The seasonality of a transmissible disease could be detected by characterizing the peak timing. While the drivers of seasonality are still under discussion, the seasonal patterns of a disease are influenced by various factors, including the pathogen, subpopulation, geographic location, climate, and human behavior. Currently, climate change is altering the patterns of disease transmission.

In this work, we construct and investigate a fundamental discrete-time model with respect to the annual sequence of cumulative incidences in epidemic seasons for seasonal illnesses. Given a seasonality in the spread of a transmissible disease, we focus on the annual sequences of cumulative incidences by successive epidemic seasons, while we do not consider the driver of the seasonality.

Assuming the simplest Kermack-McKendrick SIR model for the disease spread during each epidemic season, we determine the cumulative incidence of infection for each season. Furthermore, provided that the immunity acquired in previous epidemic seasons wanes with a probability within the interval period of successive epidemic seasons, we introduce the immune subpopulation at the onset of each epidemic season. This assumption is pivotal in constructing a recurrence relation of cumulative incidences of infection between successive epidemic seasons. With additional minimal assumptions for the modeling, our primitive mathematical model generating the annual sequence of cumulative incidences is represented by the following recurrence relation between the final proportions of immune population $w_\infty^{[n]}$ and $w_\infty^{[n+1]}$ in the total population at the conclusion of the n th and $n+1$ th epidemic seasons:

$$w_\infty^{[n+1]} + \frac{1}{\mathscr{R}_0}\log(1 - w_\infty^{[n+1]}) = \sigma w_\infty^{[n]} + \frac{1}{\mathscr{R}_0}\log(1 - v_0^{[n]} - \sigma w_\infty^{[n]}),$$

where $\mathscr{R}_0$ is the basic reproduction number for the SIR epidemic dynamics during the epidemic season. It is defined for the Kermack-McKendrick SIR model with the initial condition where there are no immune individuals at the onset of the season. $v_0^{[n]}$ is the initial proportion of infective population for the n th epidemic season, which is positive and uncertain. $\sigma \in (0,1)$ is the immunity retention rate within the interval period between successive epidemic seasons. The cumulative incidence of infection for the $n+1$ th epidemic season is given as $p_\infty^{[n+1]} = w_\infty^{[n+1]} - \sigma w_\infty^{[n]}$ based on our model. This model can be considered a specific type of discrete-time SIRS epidemic dynamics model. We investigate the mathematical characteristics of our model and compare its features with the standard SIRS epidemic dynamics model of ordinary differential equations, particularly in relation to the observed nature of the epidemic dynamics by them.