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A population dynamics model for the information spread in a community with the heterogeneity of individuality

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This talk presents a mathematical model to describe the temporal variation of the frequency of accepters for a matter spreading over a community, where the matter may be, for example, a piece of information, an innovation, a mode, a transmissible disease, etc. We shall call such a matter "information" hereafter in general. Our mathematical model is a population dynamics model taking account of the heterogeneity of individuality that characterizes the easiness/hardness for the members of the community to accept an information.

In history, we may consider the spread of an innovation or the penetration of a novel appliance as the example. In sociological science, such a process of the spread of an information was theoretically discussed in some contexts, among which Mark Granovetter's works are well-known [1, 2, 3]. The simple model used by Granovetter was sometimes called *threshold model* today, whereas, rigorously saying, it was not a population dynamics model since it was not to describe the temporal variation of the spread of an information even though it could conceptually imply the temporal variation.

In this research project, we construct and analyze a reasonable population dynamics model given by the following difference equation, of which a special case may be regarded as corresponding to the Granovetter's threshold model:

$$P_{t+1} = \left[1 + \gamma b\left\{\varphi_0 - P_t + (1-\varphi_0)F(\alpha P_t)\right\}\right] P_t,$$

where P_t means the frequency of accepters at time-step t (e.g., in the unit of day) with respect to an information in the considered community. φ_0 is the initial frequency of accepters. Parameters γ and b are positive constants, which respectively mean the probability to make the decision to accept or deny the information, and the likeliness to get a chance to make the decision.

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The value αP_t with a positive constant α is introduced as the strength of the social effect on the decision-making, where the social effect may mean the likeliness for the member to contact the information in the social situation depending on the accepter's frequency. The function F defines the frequency distribution of threshold value with respect to the decision-making among the members of the community, which reflects the heterogeneity of individuality about the decision-making. Following the idea of Granovetter's threshold model, the rule of decision-making is assumed to be governed by the following rule: the decision-making is possible for any member who has the threshold value ξ such that $\xi \leqslant \alpha P$, while it is not available for any member who has the threshold ξ such that $\xi > \alpha P$, where the threshold ξ characterizes the individuality according to the decision-making about whether a member could accept the received information or not. The value of $F(x)$ gives the frequency of members who have the threshold value ξ less than x in the community.

In this talk, we will present the modeling of the above mathematical model and discuss its mathematical features, especially focusing on how the threshold distribution in the community could affect the dynamical nature for the spread of an information over the community.

References

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