A mathematical consideration on the disease spread in a community with two activity classes

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Infectious disease are generally spread by human activity. In this work, we are investigating the estimation of social situation by using infection probabilities in the case of disease outbreak in a certain community with members who have different level of activities.

Here we proposed a model where the community members are divided into two classes based on their activity level in daily life: active and less-active. The activities take place in two phases, private and social. The active class members of the community do activities in both private and social phases each for a certain amount of time, while on the other hand the less-active class members stay at home i.e. in the private phase for the majority of their daily life. There are probabilities of infection at the private and social phase. The models for describing the expected number of new infection are as below:

$$
\begin{aligned}
E_l(\alpha,q,N) &= \beta_p(\alpha,q,N)\big(1-q\big)N \\
E_a(\alpha,q,N) &= \Big[1-\big\{1-(1-\alpha)\beta_p(\alpha,q,N)\big\}\big\{1-\alpha\beta_s(\alpha,q,N)\big\}\Big]qN,
\end{aligned}
$$

where $E_l(\alpha,q,N)$ is the expected number of new infection cases in the less-active subpopulation, $E_a(\alpha,q,N)$ is the expected number of infection cases in the active subpopulation, q is the ratio of active subpopulation in the community, $(1-q)N$ is the number of less-active individuals, qN is the number of active individuals, and α is the average ratio of time spent by the active individuals in social phase. $\beta_p(\alpha,q,N)$ and $\beta_s(\alpha,q,N)$ are the infection probabilities at private phase and social phase, respectively, as functions of α, q and N:

$$
\begin{aligned}
\beta_p(\alpha,q,N) &= \sigma_p(1-q)N + \sigma_p(1-\alpha)qN \\
\beta_s(\alpha,q,N) &= \sigma_s\alpha qN.
\end{aligned}
$$

The total number of new infection in the community is $E(\alpha,q,N) = E_l(\alpha,q,N) + E_a(\alpha,q,N)$.

We investigated the parameter dependence of $E(\alpha,q,N)$, especially the q-dependence and the α-dependence of the expected number of secondary cases in the community to gain insight on the social situations with minimum number of secondary infections. Our analysis suggest that there could be two qualitatively different situations where the expected number of the secondary infection cases are minimized. The parameters can be translated into the social characteristic of the community for the prevention of disease spread. We will also discuss what social strategy would be suitable to suppress the increase of infection.