A mathematical model for the influence of the social insensitivity on the SIS epidemic dynamics: Occurrence of oscillatory behavior

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When a transmissible disease invades, the community may respond to the disease in such a way of wearing masks or getting the vaccine to prevent the serious symptom and the disease transmission. However, the community may not necessarily show such a social response to a transmissible disease, being insensitive to the disease spread. In this work, we propose a specific Susceptible-Infective-Susceptible (SIS) model, taking account of the effect of such social response. Our model is given by the following system of ordinary differential equations:

$$\begin{aligned} \frac{dS}{dt} &= -\beta(M)SI + q(M)I \\ \frac{dI}{dt} &= \beta(M)SI - q(M)I \\ \frac{dM}{dt} &= \Gamma(I) - \mu M, \end{aligned}$$

where $M = M(t)$ is the strength of the social response at time t, which has the natural decay rate μ. $\Gamma(I)$ is called the social sensitivity function of the number of infectives I, which could reflect the cultural characteristics, the social environment, and the educational circumstance. We introduce a critical value I_c for the number of infectives below which the community is insensitive to the disease, and the function Γ of I is given by

$$\Gamma(I) := \begin{cases} 0 \quad \text{for} \quad I \leq I_c; \\ \\ \gamma(I - I_c) \quad \text{for} \quad I > I_c. \end{cases}$$

In this work, we discuss the effect of the social sensitivity and the social insensitivity on the epidemic dynamics. Our analysis shows that the population necessarily approaches an endemic state in a monotonic manner or in a damped oscillatory manner. In this work, we focus on the occurrence of such an oscillatory behavior, since the temporal oscillation of the infective population size corresponds to a repetition of outbreaks of disease spread. The results imply that the social sensitivity, the social insensitivity, and the community tolerance for maintaining the social response have a certain relevance to the repetition of outbreaks of disease spread.