An epidemic dynamics model of reinfectious disease: The influence of visitor acceptance

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In this work, we focus on the influence of temporal visitors in a community according to the endemicity of a transmissible disease spreading over the community. Making use of an SIR type of mathematical model, we consider the influence of visitors on the spread of a reinfectious disease in a community, especially assuming that a certain proportion of visitors are immune or vaccinated. The reinfectivity of disease means that the immunity gained by either vaccination or recovery is imperfect. We consider the following system of ordinary differential equations:

$$\text{Dynamics for the visitor populaton:} \begin{cases} \dfrac{dS_{\rm v}}{dt} = (1-\rho)\Lambda - \beta\dfrac{I_{\rm r}+I_{\rm v}}{N+m}S_{\rm v} - qS_{\rm v}; \\ \dfrac{dI_{\rm v}}{dt} = \beta\dfrac{I_{\rm r}+I_{\rm v}}{N+m}S_{\rm v} + \epsilon\beta\dfrac{I_{\rm r}+I_{\rm v}}{N+m}R_{\rm v} - \gamma I_{\rm v} - qI_{\rm v}; \\ \dfrac{dR_{\rm v}}{dt} = \rho\Lambda + \gamma I_{\rm v} - \epsilon\beta\dfrac{I_{\rm r}+I_{\rm v}}{N+m}R_{\rm v} - qR_{\rm v}; \end{cases}$$

$$\text{Dynamics for the resident populaton:} \begin{cases} \dfrac{dS_{\rm r}}{dt} = -\beta\dfrac{I_{\rm r}+I_{\rm v}}{N+m}S_{\rm r} - \sigma S_{\rm r}; \\ \dfrac{dI_{\rm r}}{dt} = \quad \beta\dfrac{I_{\rm r}+I_{\rm v}}{N+m}S_{\rm r} + \epsilon\beta\dfrac{I_{\rm r}+I_{\rm v}}{N+m}R_{\rm r} - \gamma I_{\rm r}; \\ \dfrac{dR_{\rm r}}{dt} = \sigma S_{\rm r} + \gamma I_{\rm r} - \epsilon\beta\dfrac{I_{\rm r}+I_{\rm v}}{N+m}R_{\rm r}, \end{cases}$$

where $S_{\rm v}$, $I_{\rm v}$, and $R_{\rm v}$ are the subpopulation sizes of susceptible, infective, and immune/vaccinated visitors respectively. $S_{\rm r}$, $I_{\rm r}$, and $R_{\rm r}$ are the corresponding subpopulation sizes about the residents. The population sizes of residents and visitors in the community are denoted by $N = S_{\rm r} + I_{\rm r} + R_{\rm r}$ and $m = S_{\rm v} + I_{\rm v} + R_{\rm v}$ respectively. Λ represents the net immigration rate of visitors. Parameter ρ is the proportion of immune visitors at the moment of their immigration ($0 \le \rho \le 1$). The proportion $1-\rho$ of immigrating visitors is susceptible. Parameter q is the per capita emigration rate of visitors. Parameter $\epsilon\beta$ is the reinfection coefficient for the immune/vaccinated individual, while β is the infection coefficient for the susceptible. Parameter γ is the recovery rate of infective individual, and the recovered individual gets the immunity. Only the susceptible residents can get the vaccination with rate σ, and it is not available for any visitor staying in the community.

We are going to discuss the condition that the acceptance of visitors could suppress or trigger the endemicity of a reinfectious disease in the community.