SIR model with the distribution of preventive behaviors in a community

予防行動分布をもつ集団におけるSIRモデル

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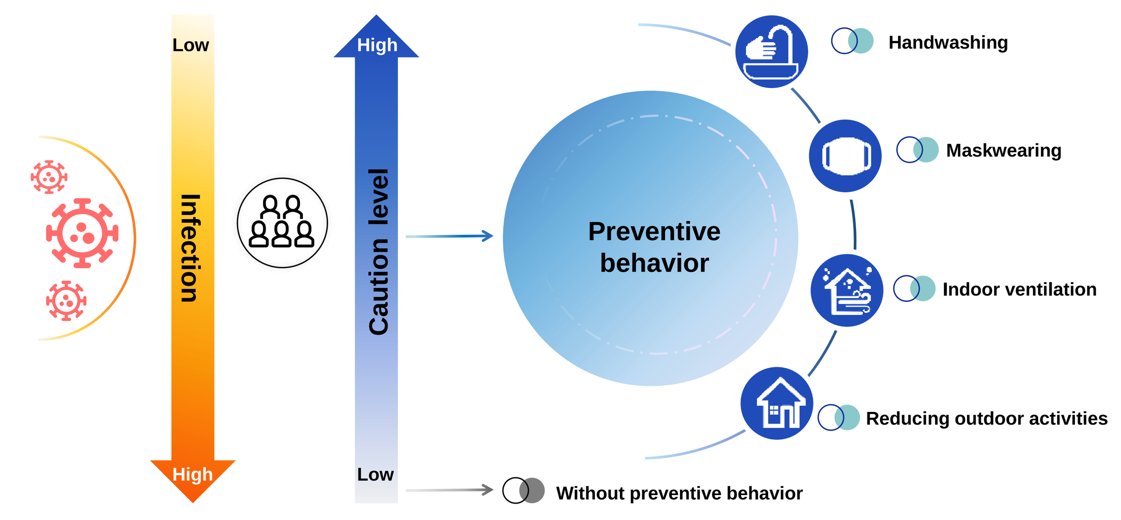
What is considered

We consider a mathematical model for the epidemic dynamics with the heterogeneity of preventive behavior among individuals about a disease transmission, focusing on the relation of the distribution of preventive behavior to the final epidemic consequence in a community. By mathematical analyses on the model, we are trying to provide some theoretical insights on the contribution of the heterogeneity in social behavior to the epidemic dynamics.

Assumptions



- Individuals are categorized into n classes based on their caution level;
- Constant total and class population sizes;
- Caution level affects preventive behavior in both susceptible and infected individuals;
- Susceptible individuals of low caution level are more likely to get infected;
- Low caution level individuals contribute more to the disease transmission due to their lower-quality preventive behavior, even after infection.

Model

$$\frac{dS_i}{dt} = -\varepsilon_i\beta\sum_{k=1}^{n}\gamma_k I_k S_i;$$

$$\frac{dI_i}{dt} = \varepsilon_i\beta\sum_{k=1}^{n}\gamma_k I_k S_i - \rho I_i;$$

$$\frac{dR_i}{dt} = \rho I_i.$$

Initial condition: $S_i(0) + I_i(0) = p_i N$, $I_i(0) > 0$ for some i, and $R_i(0) = 0$ for any i.

- S_i, I_i, and R_i: Sizes of susceptible, infective, and recovered subpopulation of caution level i respectively.
- N: Total population size in the community.
- p_i: Proportion of the class size of caution level i, where $p_i \in (0,1)$ and $\sum_{i=1}^{n} p_i = 1$.
- $S_i(t) + I_i(t) + R_i(t) = p_i N$ for any $t \geq 0$.
- ε_i: Efficiency of preventive behavior, where $\varepsilon_i \in (0,1]$ and $\varepsilon_1 > \varepsilon_2 > \ldots > \varepsilon_n$.
- $\varepsilon_i\beta$: Infection coefficient of susceptibles of caution level i, where $\beta > 0$.
- γ_k: Contribution of the infectives of caution level k to the disease transmission, where $\gamma_k \in (0,1]$ and $\gamma_1 > \gamma_2 > \ldots > \gamma_n$.
- ρ: Recovery rate.

Non-dimensional variables and parameters

$$\tau := \rho t;\quad u_i := \frac{S_i}{N};\quad v_i := \frac{I_i}{N};\quad w_i := \frac{R_i}{N};\quad \langle\varepsilon\rangle := \sum_{k=1}^{n}\varepsilon_k p_k;$$

$$\mathscr{R}_{0,i} := \frac{\gamma_i\langle\varepsilon\rangle\beta N}{\rho};\quad \mathscr{R}_0 := \frac{\langle\varepsilon\gamma\rangle\beta N}{\rho};\quad \mathscr{R}_0^{\mathrm{sup}} := \frac{\beta N}{\rho};$$

$u_i(0) + v_i(0) = p_i$; $v_i(0) > 0$ for some i; $w_i(0) = 0$ for any i.

$$\frac{du_i}{d\tau} = -u_i\frac{\varepsilon_i}{\langle\varepsilon\rangle}\sum_{k=1}^{n}\mathscr{R}_{0,k}v_k;$$

$$\frac{dv_i}{d\tau} = u_i\frac{\varepsilon_i}{\langle\varepsilon\rangle}\sum_{k=1}^{n}\mathscr{R}_{0,k}v_k - v_i;$$

$$\frac{dw_i}{d\tau} = v_i.$$

Final epidemic size $W_\infty := 1 - \sum_{i=1}^{n} u_i^\infty$

$$u_i^\infty := \lim_{\tau\to\infty} u_i(\tau) = u_i(0)U_\infty^{\varepsilon_i};$$

$$W_\infty = \sum_{i=1}^{n} w_i^\infty = \sum_{i=1}^{n}(p_i - u_i^\infty),$$

where $U_\infty \in (0,1)$ is the unique root of equation $\log U^{\langle\varepsilon\rangle} + \sum_{k=1}^{n}\mathscr{R}_{0,k}[p_k - u_k(0)U^{\varepsilon_k}] = 0$.

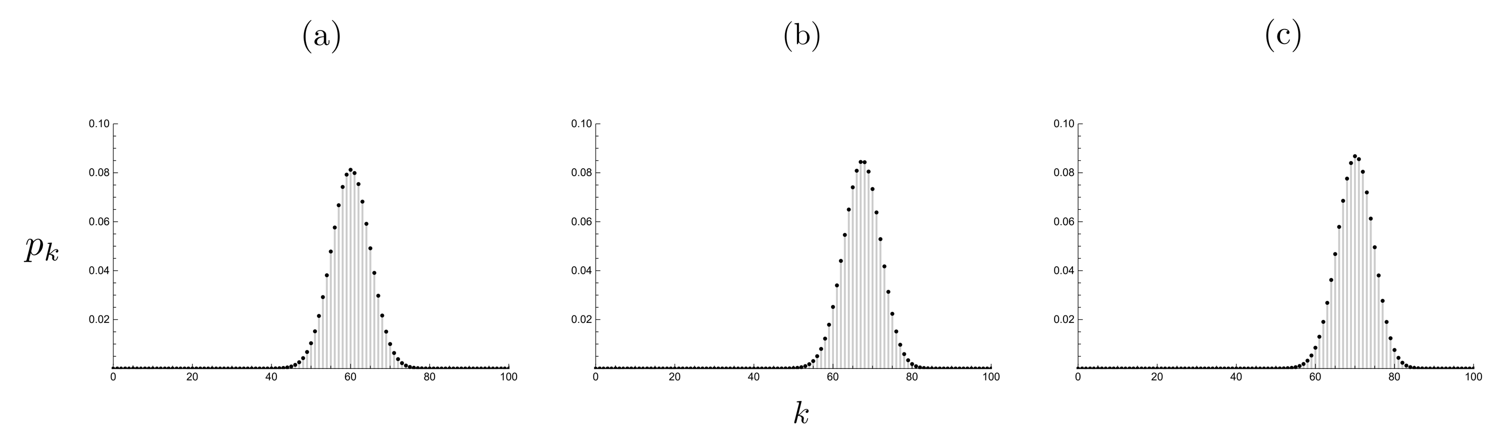
Infimum of the final epidemic size W^*

$$W^* := \inf_{\{u_i(0)\}} W_\infty = \sum_{i=1}^{n} w_i^* \text{ with } w_i^* := p_i - (U^*)^{\varepsilon_i}p_i,$$

where $U^* \in (0,1]$ is the smallest root of equation $\log U^{\langle\varepsilon\rangle} + \sum_{k=1}^{n}\mathscr{R}_{0,k}p_k\,(1 - U^{\varepsilon_k}) = 0$.

We find that $W^* = 0$ if $\mathscr{R}_0 \leq 1$, and $W^* > 0$ if $\mathscr{R}_0 > 1$.

A specific class size distribution



(a) $q = 0.6$; (b) $q = 0.673$; (c) $q = 0.7$; $n = 100$.

$$p_k = \frac{{}_nC_k q^k(1-q)^{n-k}}{1-(1-q)^n};$$

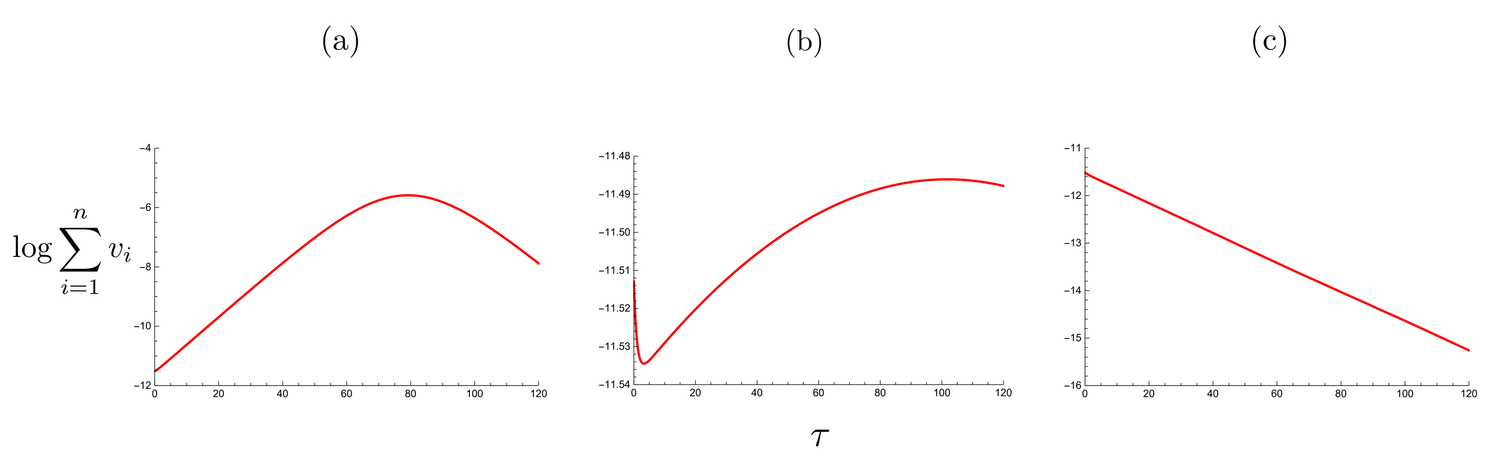
$$\varepsilon_k = b_\varepsilon^{k-1};$$

$$\gamma_k = b_\gamma^{k-1};$$

$$b_\varepsilon, b_\gamma \in (0,1);$$

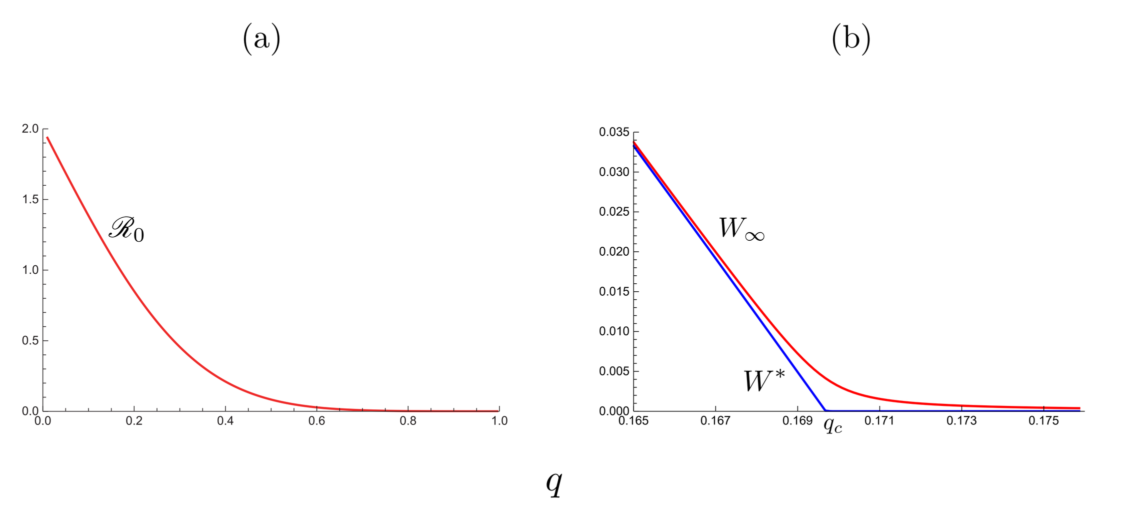
$$\theta = b_\varepsilon b_\gamma.$$

Outbreak, revival, or unsuccessful spread



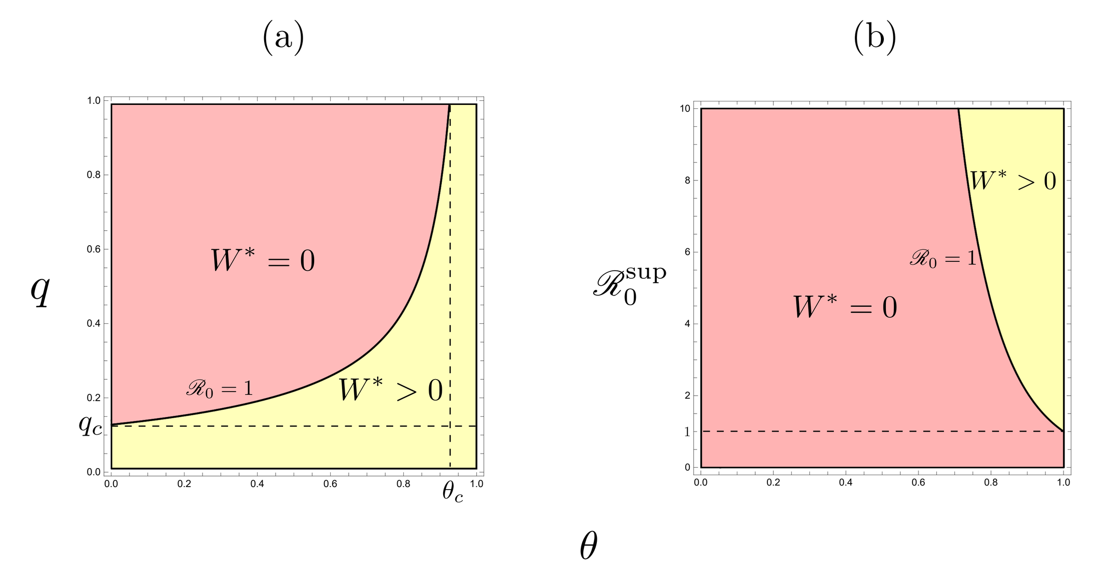
$N = 1.0\times10^5$; $n = 100$; $(b_\varepsilon, b_\gamma, \theta) = (0.990, 0.998, 0.988)$; $\beta = 2.0\times10^{-5}$; $\rho = 0.9$. $v_{80}(0) = 1.0\times10^{-5}$; $v_i(0) = 0.0$ for $i \neq 80$. $(\mathscr{R}_0, \langle\varepsilon\rangle, \langle\gamma\rangle) =$ (a) $(1.093, 0.553, 0.889)$; (b) $(1.001, 0.514, 0.876)$; (c) $(0.969, 0.500, 0.871)$.

q-dependence of $\mathscr{R}_0$ and W_∞



$N = 1.0\times10^5$; $n = 10$; $(b_\varepsilon, b_\gamma, \theta) = (0.5, 0.6, 0.3)$; $\beta = 1.0\times10^{-5}$; $\rho = 0.5$; $q_c = 0.170$; $v_1(0) = 1.0\times10^{-5}$; $v_i(0) = 0.0$ for $i \neq 1$.

Criticality of W^*



(a) $\mathscr{R}_0^{\mathrm{sup}} = 2.0$; (b) $q = 0.8$; $n = 10$; $\theta_c = 0.926$; $q_c = 0.128$.

Concluding remarks

- Final epidemic size strongly depends on the distribution of caution level among individuals in the community. There exists a criticality for the distribution of caution level that determines the severity of social damage by the disease spread.
- The larger proportion of low-cautious individuals in the community not only accelerates the spread of a transmissible disease in the early stage but also contributes to the greater final epidemic size, thereby resulting in the severer social damage.