An epidemic dynamics model with social classes different in the preventive behavior

予防行動に違いのある社会階層構造をもつ感染症伝染ダイナミクスモデル

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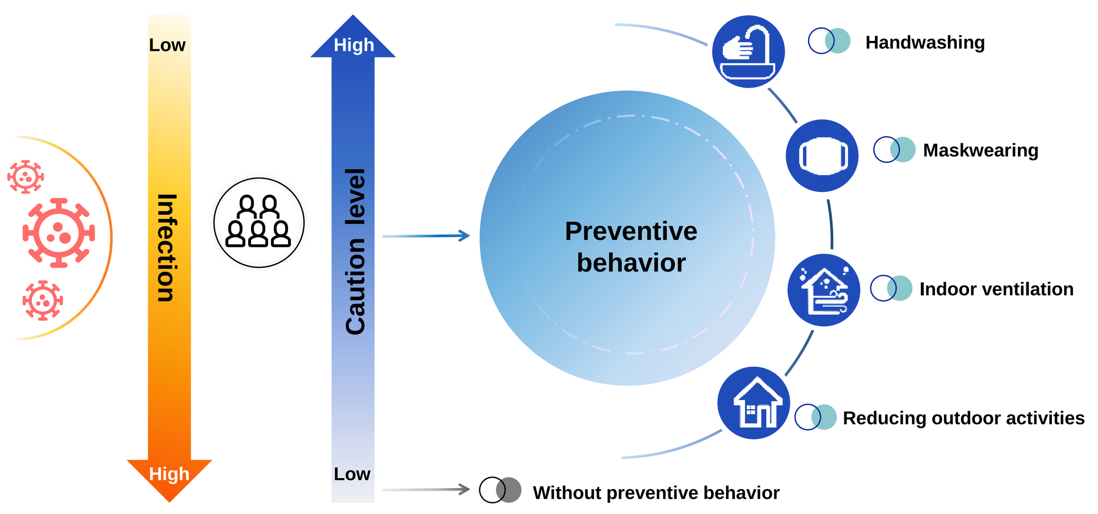
What is considered

We consider a mathematical model for the epidemic dynamics with the heterogeneity of preventive behavior among individuals about a disease transmission, focusing on the relation of the distribution of preventive behavior to the final epidemic consequence in a community. By mathematical analyses on the model, we are going to provide some theoretical results on the contribution of the heterogeneity in social behavior to the epidemic dynamics.

Assumptions



- Constant population size;
- Individuals are categorized into n classes based on their caution level;
- Caution level affects preventive behavior in both susceptible and infected individuals;
- Susceptible individuals of low caution level are more likely to get infected;
- Low caution level individuals contribute more to disease transmission due to lower-quality preventive behavior, even after infection.

Model

$$\frac{dS_i}{dt} = -\varepsilon_i \beta \sum_{k=1}^{n} c_k I_k S_i;$$

$$\frac{dI_i}{dt} = \varepsilon_i \beta \sum_{k=1}^{n} c_k I_k S_i - \rho I_i;$$

$$\frac{dR_i}{dt} = \rho I_i,$$

Initial condition: $S_i(0) + I_i(0) = p_i N$, $I_i(0) > 0$, and $R_i(0) = 0$ for $\forall i$.

- S_i, I_i, and R_i: Sizes of susceptible, infective, and recovered subpopulation of caution level i respectively.
- N: Total population size in the community.
- p_i: Proportion of subpopulation size of caution level i.
- $S_i(t) + I_i(t) + R_i(t) = p_i N$ for $\forall t \geq 0$, with $p_i \in (0,1)$, and $\sum_{i=1}^{n} p_i = 1$.
- $\varepsilon_i \beta$: Infection coefficient of susceptibles of caution level i.
- ε_i: Efficiency of preventive behavior, where $\varepsilon_i \in (0,1)$, and $\varepsilon_1 > \varepsilon_2 > \ldots > \varepsilon_n$.
- c_k: Contribution of the infectives of caution level k to the disease transmission, where $c_k \in (0,1)$, and $c_1 > c_2 > \ldots > c_n$.
- ρ: Recovery rate.

Non-dimensionalization

$$\tau := \rho t; \quad u_i := \frac{S_i}{N}; \quad v_i := \frac{I_i}{N}; \quad w_i := \frac{R_i}{N};$$

$$\langle \varepsilon \rangle = \sum_{k=1}^{n} \varepsilon_k p_k; \quad \mathscr{R}_{0i} := \frac{c_i \beta}{\rho} \langle \varepsilon \rangle N; \quad \eta_k := \frac{c_k}{\langle \varepsilon \rangle};$$

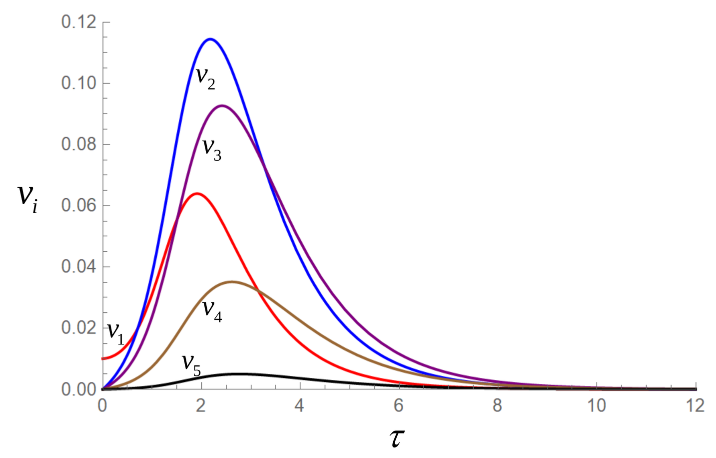
$$\frac{du_i}{d\tau} = -u_i \mathscr{R}_{0i} \frac{\varepsilon_i}{c_i} \sum_{k=1}^{n} \eta_k v_k;$$

$$\frac{dv_i}{d\tau} = u_i \mathscr{R}_{0i} \frac{\varepsilon_i}{c_i} \sum_{k=1}^{n} \eta_k v_k - v_i;$$

$$\frac{dw_i}{d\tau} = v_i,$$

$u_i(0) + v_i(0) = p_i$, $v_i(0) > 0$, $w_i(0) = 0$ for $\forall i$.

Temporal variation



$N = 1.0 \times 10^5$; $n = 5$; $p_k = [{}_nC_k q^k (1-q)^{n-k}]/[1-(1-q)^n]$; $q = 0.5$; $\varepsilon_k = \exp[-0.5(k-1)]$; $c_k = \exp[-0.1(k-1)]$; $\beta = 3.0 \times 10^{-5}$; $\rho = 0.4$; $\mathfrak{R}_0 = 3.86978$; $\langle \varepsilon \rangle = 0.515971$; $u_i(0) + v_i(0) = p_i$; $v_1(0) = 0.01$; $v_i(0) = 0.0$ for $i \neq 1$.

Conserved quantities

$$F(U(\tau)) := \frac{\ln U(\tau)}{\mathfrak{R}_0} + \sum_{k=1}^{n} \eta_k \left[p_k - u_k(0) \{U(\tau)\}^{\varepsilon_k} \right]$$

$$= \sum_{k=1}^{n} \eta_k v_k(\tau)$$

with $\mathfrak{R}_0 := \beta \langle \varepsilon \rangle N / \rho$;

$$\left\{ \frac{u_i(\tau)}{u_i(0)} \right\}^{1/\varepsilon_i} = \left\{ \frac{u_j(\tau)}{u_j(0)} \right\}^{1/\varepsilon_j} = U(\tau)$$

for $\forall i, j$.

Final epidemic size $w_\infty := 1 - \sum_{i=1}^{n} u_i^\infty$

$$w_\infty = \sum_{i=1}^{n} w_i^\infty \quad \text{with} \quad w_i^\infty := p_i - u_i^\infty,$$

where $U_\infty \in (0,1)$ is the root of equation $F(U) = 0$, and $u_i^\infty = u_i(0) U_\infty^{\varepsilon_i}$.

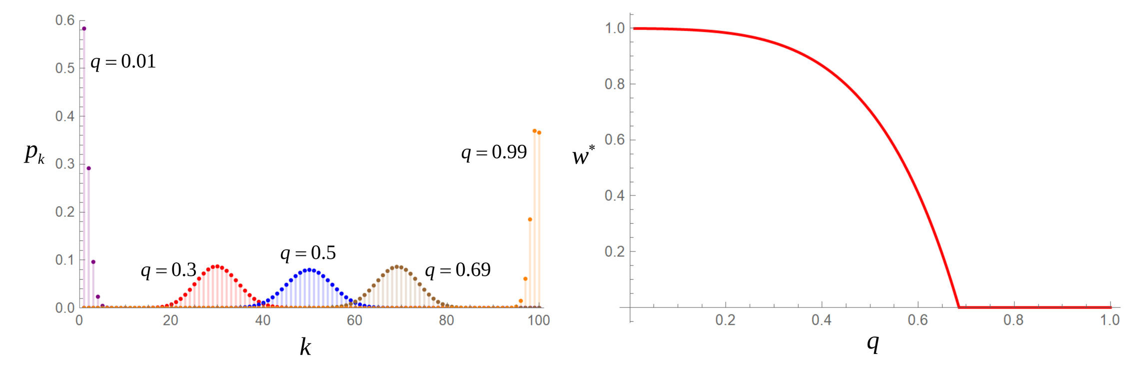
Infimum of the final epidemic size w^*

$$w^* := \inf_{\{u_i(0)\}} w_\infty = \sum_{i=1}^{n} w_i^* \text{ with } w_i^* := p_i \{1 - (U^*)^{\varepsilon_i}\},$$

where $U^* \in (0,1]$ is the root of equation

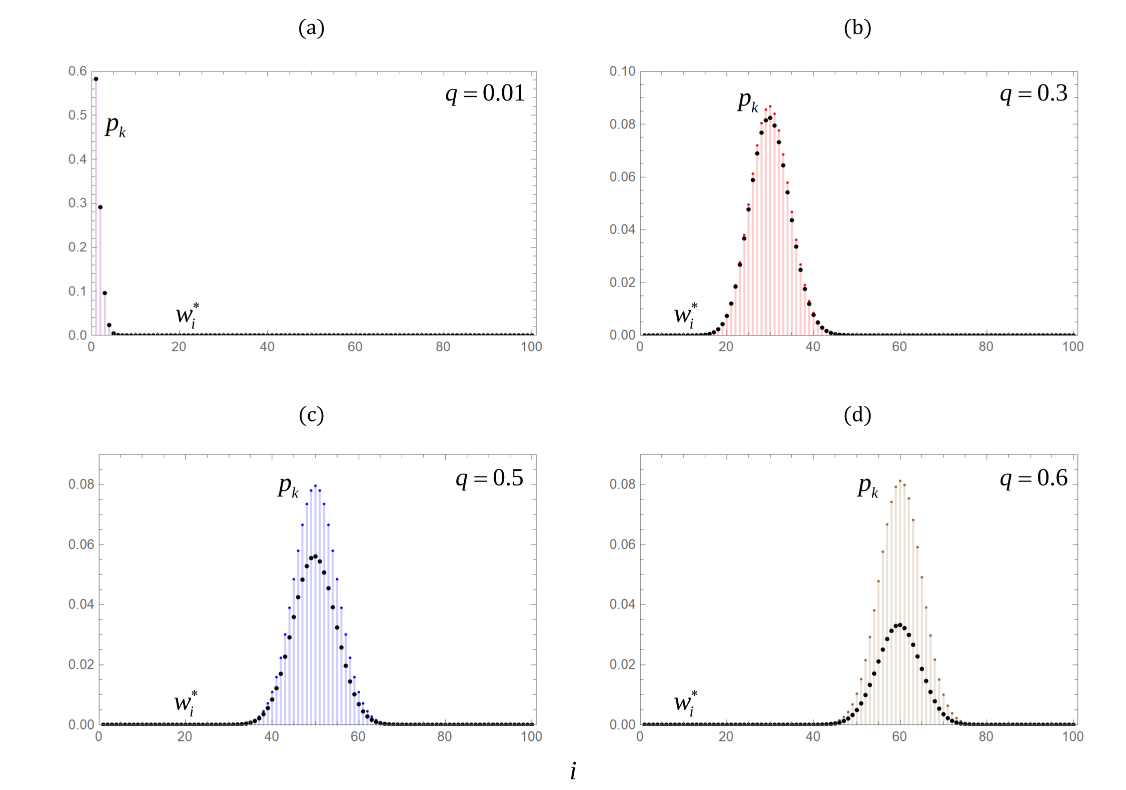
$$\frac{\ln U}{\mathfrak{R}_0} + \sum_{k=1}^{n} \eta_k p_k \{1 - (U)^{\varepsilon_k}\} = 0.$$

Relation to the class size distribution



$N = 1.0 \times 10^5$; $n = 100$; $p_k = [{}_nC_k q^k (1-q)^{n-k}]/[1-(1-q)^n]$; $\varepsilon_k = \exp[-0.02(k-1)]$; $c_k = \exp[-0.01(k-1)]$; $\beta = 3.0 \times 10^{-5}$; $\rho = 0.4$.

Distribution of final sizes



$N = 1.0 \times 10^5$; $n = 100$; $p_k = [{}_nC_k q^k (1-q)^{n-k}]/[1-(1-q)^n]$; $\varepsilon_k = \exp[-0.02(k-1)]$; $c_k = \exp[-0.01(k-1)]$; $\beta = 3.0 \times 10^{-5}$; $\rho = 0.4$.

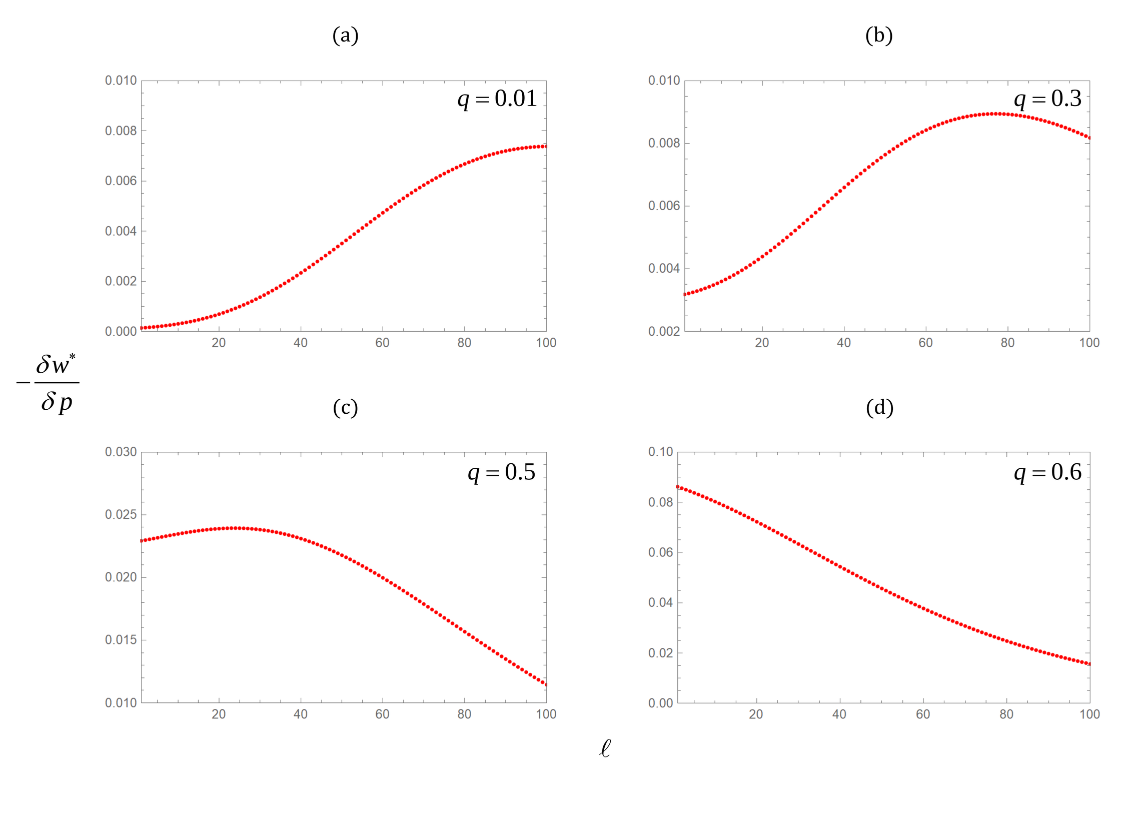
Sensitivity to the variation of class size

For a chosen ℓ,

$$p_\ell \to p_\ell - \delta p;$$
$$p_{\ell-1} \to p_{\ell-1} + \alpha \delta p;$$
$$p_{\ell+1} \to p_{\ell+1} + (1-\alpha)\delta p,$$

where $\alpha \in [0,1]$, $\delta p > 0$. Then $w^* \to w^* + \delta w^*$.

We find that $\delta w^* < 0$ for sufficiently small α, while $\delta w^* > 0$ for α sufficiently near to 1.



$\alpha = 0$; $N = 1.0 \times 10^5$; $n = 100$; $p_k = [{}_nC_k q^k (1-q)^{n-k}]/[1-(1-q)^n]$; $\varepsilon_k = \exp[-0.02(k-1)]$; $c_k = \exp[-0.01(k-1)]$; $\beta = 3.0 \times 10^{-5}$; $\rho = 0.4$.

Concluding remarks

- Final epidemic size strongly depends on the distribution of caution level among individuals in the community. There exists a criticality for the distribution of caution level that determines the minimum epidemic size.
- Education targeted to the largest class or the class of lower caution level may not be the most effective. The program for such an education must account for both class size distribution and difference in caution level.