A Population Dynamics Model of Two Parasite Species Competing for the Common Host with Different Parasitism Stages

共通の宿主について競争関係にある寄生者 2 種の寄生対象生育段階が異なる場合の個体群動態モデル

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Today numerous species are experiencing habitat shifts potentially driven by climate change. Invasion of an alien parasite species into a native host-parasite system could lead to substantial ecological consequences. In this work we consider a discrete time model on the competition dynamics between two parasite populations sharing the common host. We examine a scenario where a native host-parasite system is invaded by an alien parasite and we will try to discuss how the difference of parasitism stage could be related to the consequence of the competition, especially with respect to the invasion success/failure of alien parasite.

Assumptions

1. Adults die out after the reproduction season.
2. Two parasite species are specialists with a common host.
3. The host parasitized by one parasite species is not parasitized by the other parasite species.
4. The parasitism by two parasite species targets the stage of juvenile and adult respectively.

Model with alien parasite

$$H_{n+1} = e^{-a_1 P_n} \frac{r_0}{1 + e^{-a_2 Q_n} H_n/\beta} e^{-a_2 Q_n} H_n;$$
$$P_{n+1} = \rho_1 (1 - e^{-a_1 P_n}) \frac{r_0}{1 + e^{-a_2 Q_n} H_n/\beta} e^{-a_2 Q_n} H_n;$$
$$Q_{n+1} = \rho_2 (1 - e^{-a_2 Q_n}) H_n.$$

- H_n, P_n, and Q_n : Host and two parasite populations
- a_1 and a_2 : Coefficients for the successful parasitism
- r_0 : Supremum for the expected number of host offspring produced by an adult host
- β : Coefficient of the strength of the intraspecific density effect on host reproduction
- ρ_1 and ρ_2 : Expected number of parasite offspring born per parasitized host

The non-dimensionalised system:

$$h_{n+1} = e^{-p_n} \frac{r_0}{1 + e^{-q_n} h_n} e^{-q_n} h_n;$$
$$p_{n+1} = \alpha_1 (1 - e^{-p_n}) \frac{r_0}{1 + e^{-q_n} h_n} e^{-q_n} h_n;$$
$$q_{n+1} = \alpha_2 (1 - e^{-q_n}) h_n.$$

$h_n := \frac{H_n}{\beta}$; $p_n := a_1 P_n$; $q_n := a_2 Q_n$; $\alpha_1 := a_1 \rho_1 \beta$; $\alpha_2 := a_2 \rho_2 \beta$

Native host-parasite system

Model J: The parasite targets the juvenile stage.

$$h_{n+1} = e^{-p_n} \frac{r_0}{1 + h_n} h_n;$$
$$p_{n+1} = \alpha_1 (1 - e^{-p_n}) \frac{r_0}{1 + h_n} h_n.$$

Model A: The parasite targets the adult stage.

$$h_{n+1} = \frac{r_0}{1 + e^{-q_n} h_n} e^{-q_n} h_n;$$
$$q_{n+1} = \alpha_2 (1 - e^{-q_n}) h_n.$$

Theorem 1

When $r_0 \le 1$, the host population necessarily goes extinct, and so does the parasite population.

Theorem 2

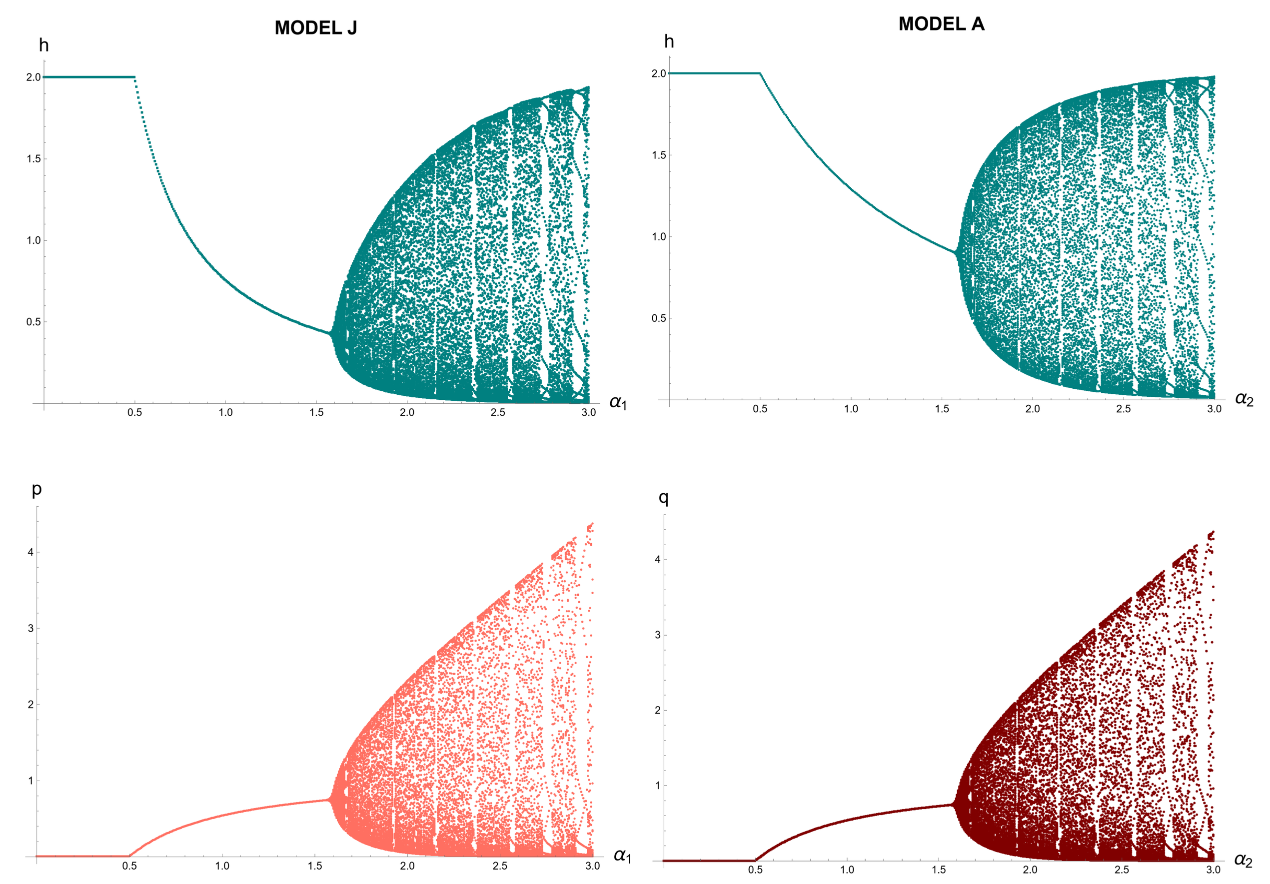
When $r_0 > 1$, the parasite extinction equilibrium E_0 is

$$\begin{cases} \text{locally asymptotically stable if } r_0 < 1 + \frac{1}{\alpha}; \\ \text{unstable if } r_0 > 1 + \frac{1}{\alpha}. \end{cases}$$

Lemma

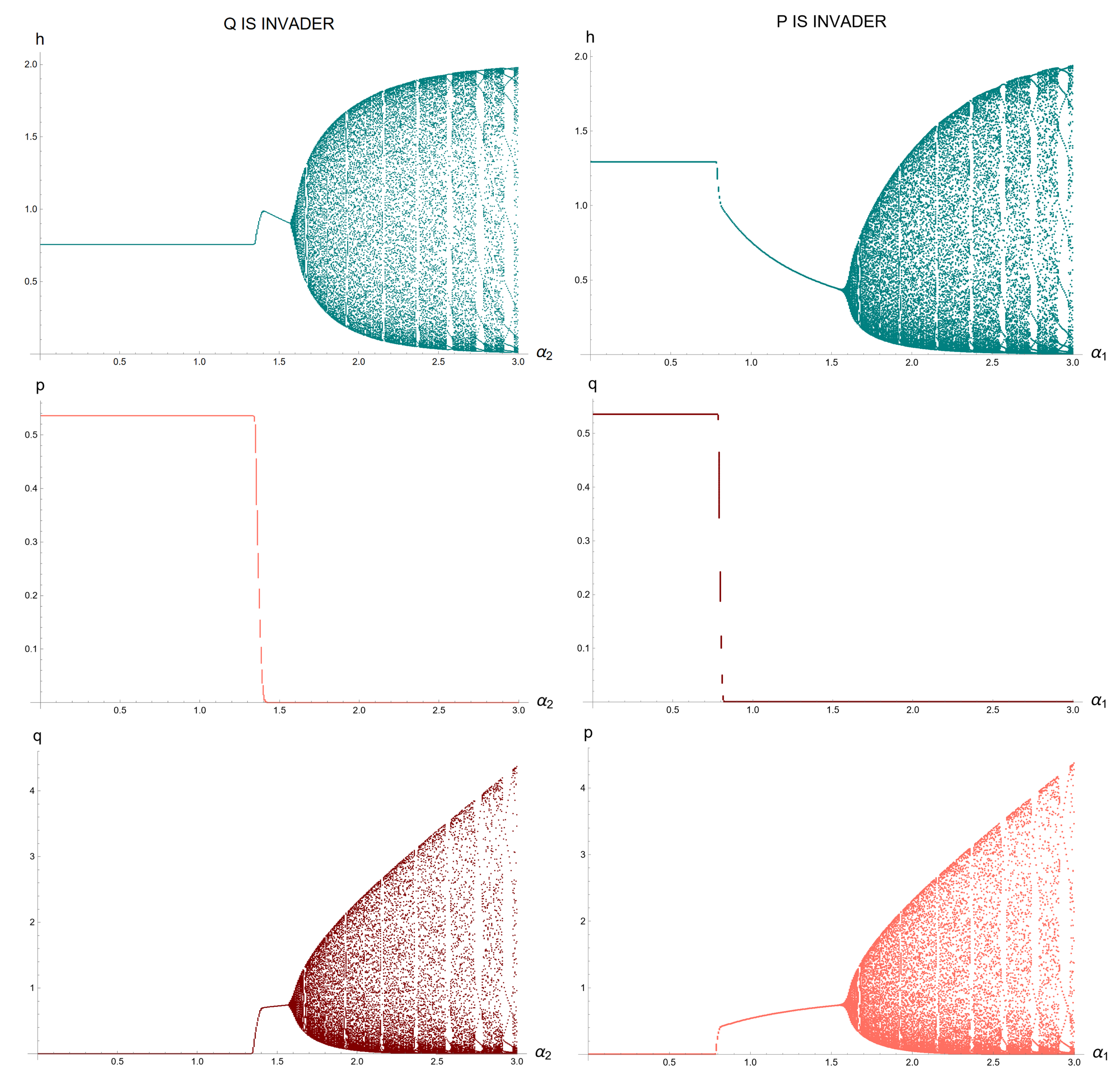
For Model J and A, the coexistent equilibrium E_+ uniquely exists if and only if $r_0 > 1 + \frac{1}{\alpha}$.

Bifurcation for native host-parasite system



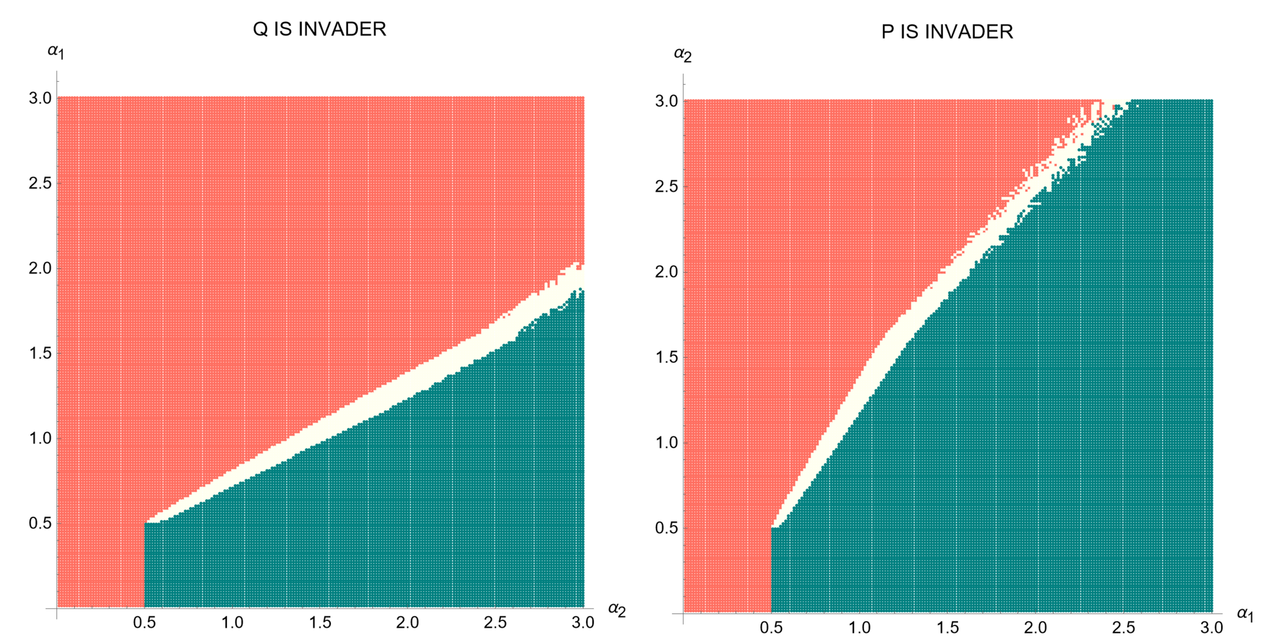
$r_0 = 3.0$

Bifurcation for model with alien parasite



$h_0 = 2.0$; $p_0 = 0.00001$; $q_0 = 0.01$; $r_0 = 3.0$; $\alpha_1 = 1.0$ $h_0 = 2.0$; $p_0 = 0.01$; $q_0 = 0.00001$; $r_0 = 3.0$; $\alpha_2 = 1.0$

Invasion success/failure



$h_0 = 2.0$; $p_0 = 0.00001$; $q_0 = 0.01$; $r_0 = 3.0$; $\alpha_1 = 1.0$ $h_0 = 2.0$; $p_0 = 0.01$; $q_0 = 0.00001$; $r_0 = 3.0$; $\alpha_2 = 1.0$

Concluding Remarks

Invasion success is significantly depending on which stage is targeted by alien parasite. Targeting the juvenile host over the adult host appears to have an advantage in the invasion success. It is much likely to exclude the native parasite whereas the coexistence is rarely possible. However, if we consider invasion of 2 parasites with same target then this may not be the case.