A Sedentary Population Dynamics Model in Fragmented Habitat: Local Extinction by Global Density Effect

分断された生息域における定住性個体群動態モデル：大域的密度効果による局所的絶滅

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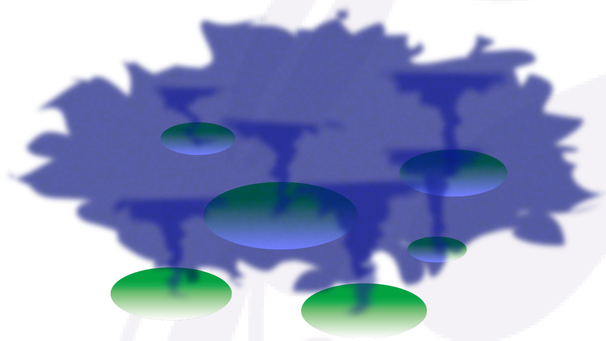
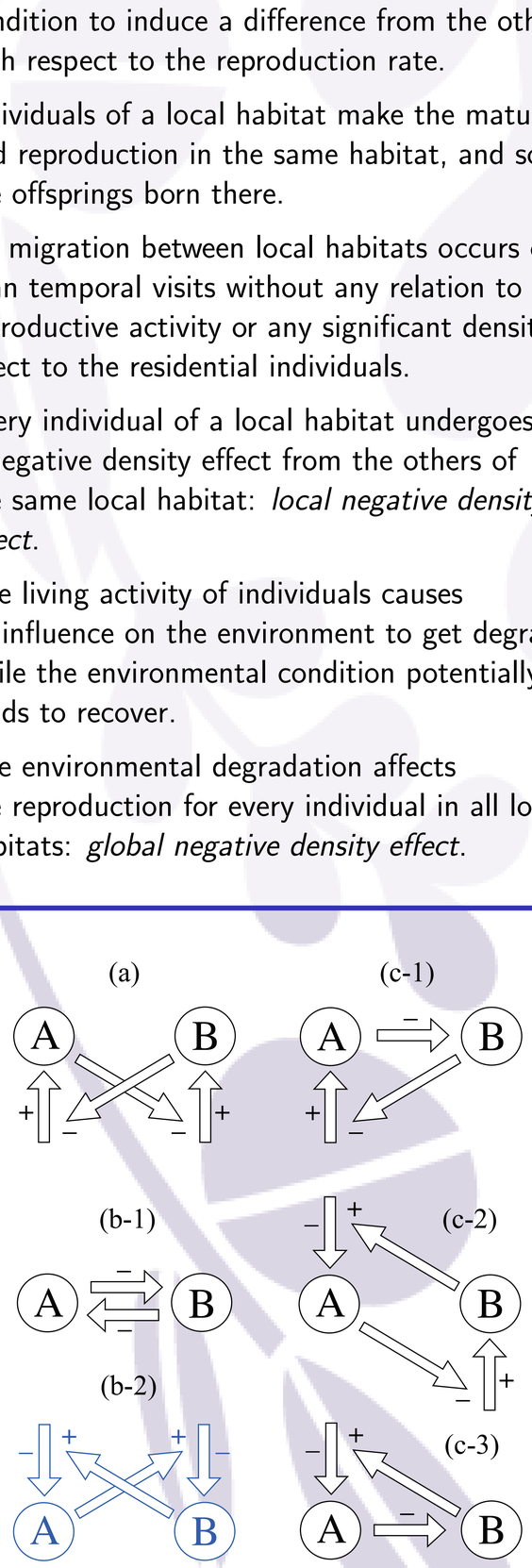
In contrast to the interaction between individuals directly affecting the population dynamics, there could be an indirect interaction between individuals of different local habitats even for the sedentary population. We shall consider a single species population dynamics model for such a sedentary population in a fragmented habitat with patchy local habitats. Our results by analyzing it imply the possibility of the extinction of a local population due to the global density effect. Further we consider how the heterogeneity of local environments in local habitats is related to the equilibrium population size in the habitat, and try to discuss the better or worse management of the habitat design for the population conservation.

Assumptions



1. A single species population inhabits a region.
2. The habitat region is fragmented to be composed with a number of local habitats, i.e., habitual patches with the reproduction-suitable condition.
3. Each local habitat may have the environmental condition to induce a difference from the other with respect to the reproduction rate.
4. Individuals of a local habitat make the maturation and reproduction in the same habitat, and so do the offsprings born there.
5. No migration between local habitats occurs other than temporal visits without any relation to reproductive activity or any significant density effect to the residential individuals.
6. Every individual of a local habitat undergoes a negative density effect from the others of the same local habitat: *local negative density effect*.
7. The living activity of individuals causes an influence on the environment to get degraded, while the environmental condition potentially tends to recover.
8. The environmental degradation affects the reproduction for every individual in all local habitats: *global negative density effect*.



Model

$$\frac{dN_i}{dt} = \{g_i(N_i) - \sigma_i D\}N_i \qquad (i = 1, 2, \ldots, m)$$

$$\frac{dD}{dt} = -\rho D + \sum_{j=1}^{m} \gamma_j N_j$$

- m : Number of local habitats.
- N_i : Population size of local habitat i.
- g_i : Per capita growth rate of population in local habitat i without the global density effect.
  - $g_i(x)$ is strictly decreasing and continuous for $x \geq 0$, and differentiable for $x > 0$;
  - $g_i(0) = r_i > 0$;
  - $g_i(K_i) = 0$ for a positive value $K_i > 0$.
- D : Strength of the global density effect.
- σ_i : Coefficient of the sensitivity to the global density effect for the individual of local habitat i.
- γ_j : Coefficient of the contribution of local population in local habitat j to the global density effect.
- ρ : Recovery rate of environmental condition.

▷ **Initial condition:**

$$0 < N_i(0) \leq K_i; \quad D(0) = 0.$$

▷ **Numbering of local habitats:**

$$\frac{r_1}{\sigma_1} \geq \frac{r_2}{\sigma_2} \geq \cdots \geq \frac{r_m}{\sigma_m}$$

without loss of generality.

Theorem

Globally asymptotically stable is the equilibrium $E^*_{[s]}$

$$(N_1, N_2, \ldots, N_m, D) = (N^*_{[s],1}, N^*_{[s],2}, \ldots, N^*_{[s],s}, \underbrace{0, \ldots, 0}_{m-s}, D^*_{[s]})$$

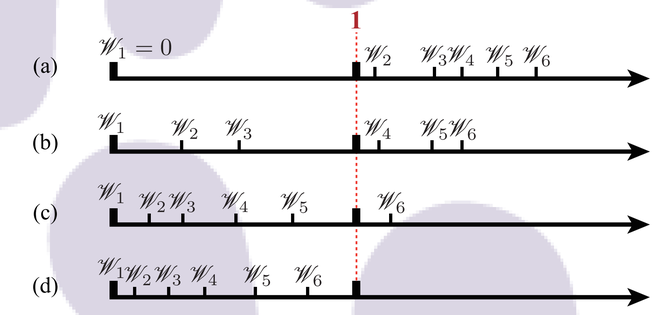
with the number of persistent local populations s such that

$$s := \max\{\ell \in \{1, 2, \ldots, m\} \mid \mathscr{W}_\ell < 1\},$$

where

$$N^*_{[s],i} = g_i^{-1}(\sigma_i D^*_{[s]}); \quad D^*_{[s]} = \frac{1}{\rho}\sum_{j=1}^{s} \gamma_j g_j^{-1}(\sigma_j D^*_{[s]});$$

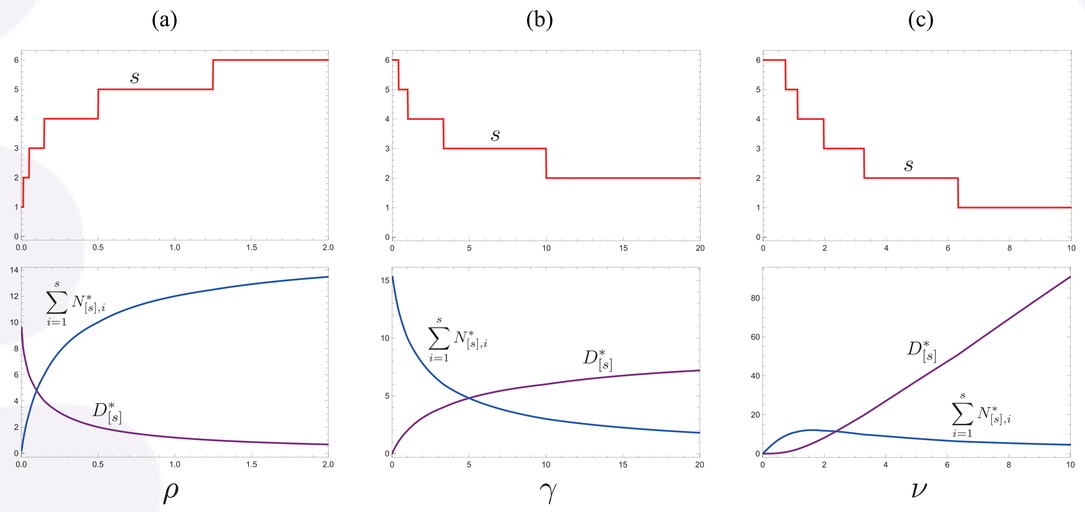
$$\mathscr{W}_\ell := \frac{1}{\rho}\sum_{j=1}^{\ell} \gamma_j \frac{\sigma_\ell}{r_\ell} g_j^{-1}\Big(\frac{r_\ell/\sigma_\ell}{r_j/\sigma_j} r_j\Big).$$



Corollary

Local habitats k_1 and k_2 with $r_{k_1}/\sigma_{k_1} = r_{k_2}/\sigma_{k_2}$ have a common destiny on the persistence of their populations: They persist or alternatively go extinct together.

Parameter dependence of asymptotically stable equilibrium

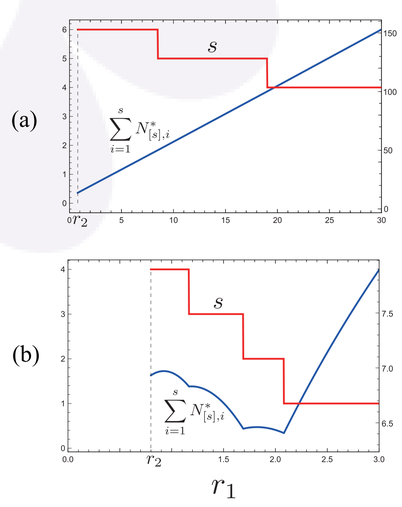


Numerical example with $g_i(N_i) = r_i - \beta_i N_i$ with $K_i = r_i/\beta_i$. (a) ρ-dependence; (b) γ-dependence with $\gamma_i = \gamma$; (c) ν-dependence with $\gamma_i = \nu r_i$. For (a), $\gamma_i = \gamma = 0.1$. For (b) and (c), $\rho = 5.0$. Commonly, $m = 6$; $\{r_i\} = \{1.0, 0.8, 0.6, 0.4, 0.2, 0.1\}$; $\beta_i = \beta = 0.2$; $\sigma_i = \sigma = 0.1$.

$$N^*_{[s],i} = \frac{\sigma_i}{\beta_i}\Big(\frac{r_i}{\sigma_i} - D^*_{[s]}\Big);$$

$$D^*_{[s]} = \frac{(1/\rho)\sum_{j=1}^{s} \gamma_j K_j}{1 + (1/\rho)\sum_{j=1}^{s} \gamma_j K_j (\sigma_j/r_j)}$$

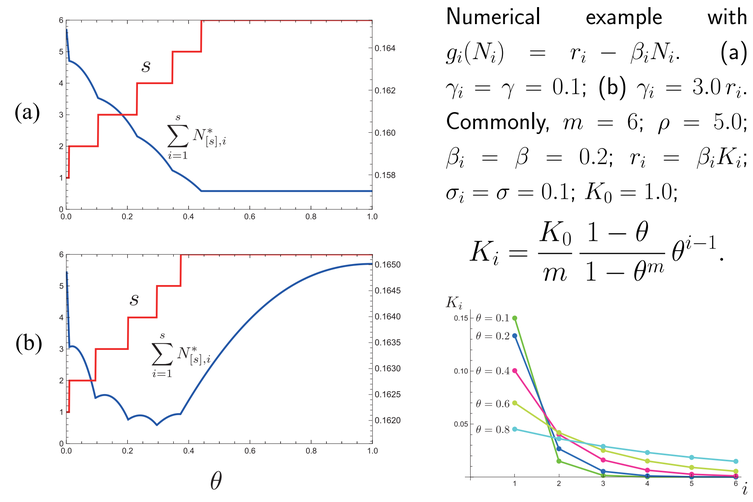
Dependence on the primary local population



Numerical example with $g_i(N_i) = r_i - \beta_i N_i$. (a) $\gamma_i = \gamma = 0.1$; (b) $\gamma_i = 3.0\, r_i$. Commonly, $m = 6$; $\rho = 5.0$; $\{r_i\} = \{r_1, 0.8, 0.6, 0.4, 0.2, 0.1\}$; $\beta_i = \beta = 0.2$; $\sigma_i = \sigma = 0.1$.

Increase in the intrinsic growth rate r_1 of local population 1 may induce the extinction of some other local population.

Heterogeneity of local habitats



Numerical example with $g_i(N_i) = r_i - \beta_i N_i$. (a) $\gamma_i = \gamma = 0.1$; (b) $\gamma_i = 3.0\, r_i$. Commonly, $m = 6$; $\rho = 5.0$; $\beta_i = \beta = 0.2$; $r_i = \beta_i K_i$; $\sigma_i = \sigma = 0.1$; $K_0 = 1.0$;

$$K_i = \frac{K_0}{m}\frac{1-\theta}{1-\theta^m}\theta^{i-1}.$$

- Indirectly influencing negative density effect may cause the extinction of some local populations where the local population has a low reproductivity and high vulnerability to the environmental degradation.
- Extinction of local populations is more likely to occur for the higher heterogeneity of local habitats, where the favorable local habitat works as the nuisance for the reproduction of less favorable ones.
- The restoration of environmental condition is effective to suppress the shrinkage of habitat region with the extinction of some local populations.
- The biological management to enrich a local environment to promote the reproduction may raise the extinction risk of the other local population.

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