Population dynamics modeling on the invasion of alien competitor

— Some remarks on the reasonability —

瀬野裕美
Hiromi Seno

東北大学大学院情報科学研究科
Graduate School of Information Sciences, Tohoku University, Sendai, Japan

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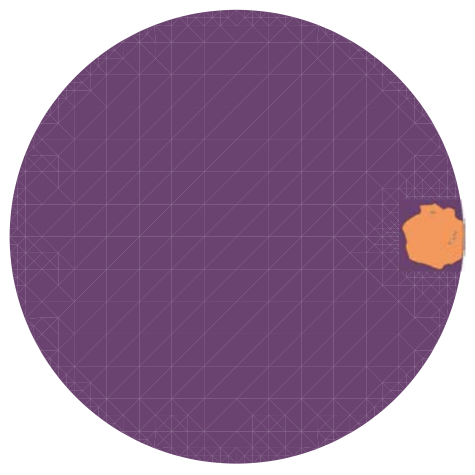
Prologue



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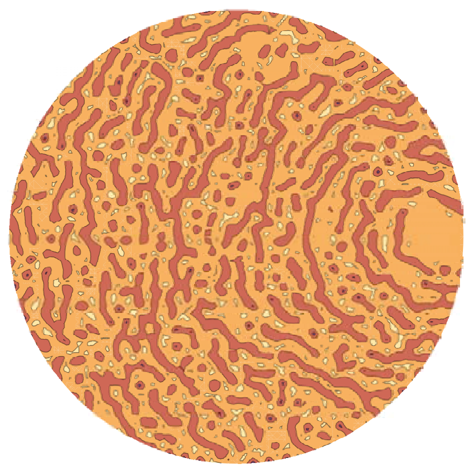

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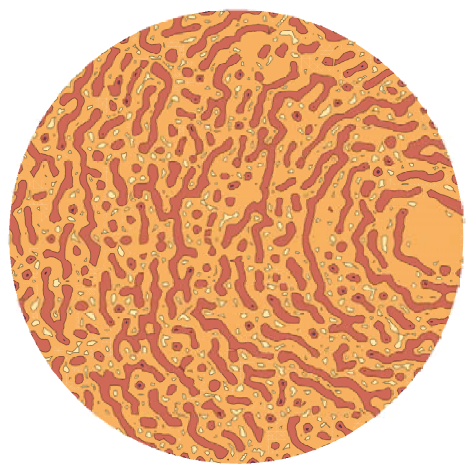

Lotka-Volterra competition system
with diffusion

$$\begin{cases} \dfrac{\partial N_1}{\partial t} = D_1 \nabla^2 N_1 + (r_1 - \beta_1 N_1 - \gamma_{12} N_2) N_1 \\ \dfrac{\partial N_2}{\partial t} = D_2 \nabla^2 N_2 + (r_2 - \beta_2 N_2 - \gamma_{21} N_1) N_2 \end{cases}$$

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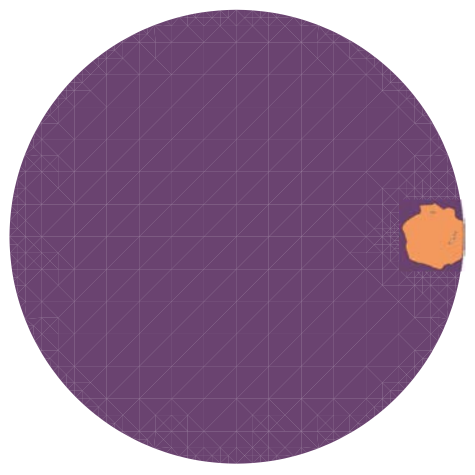

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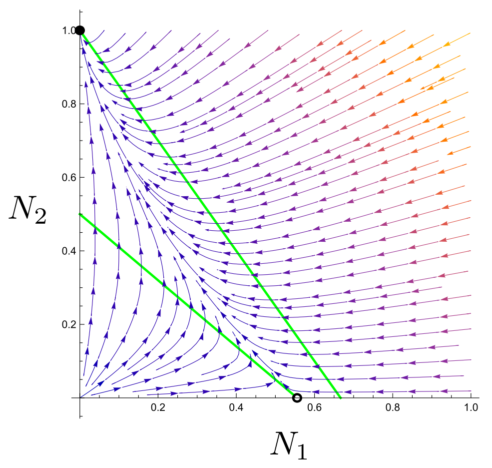

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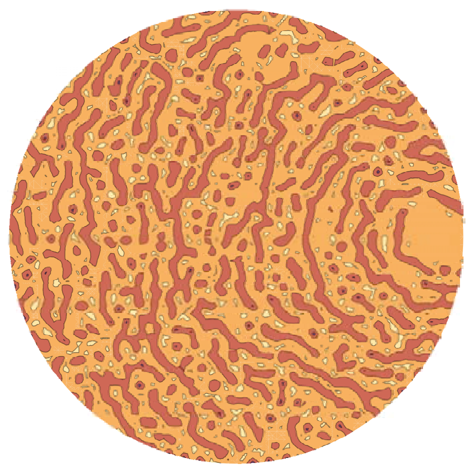

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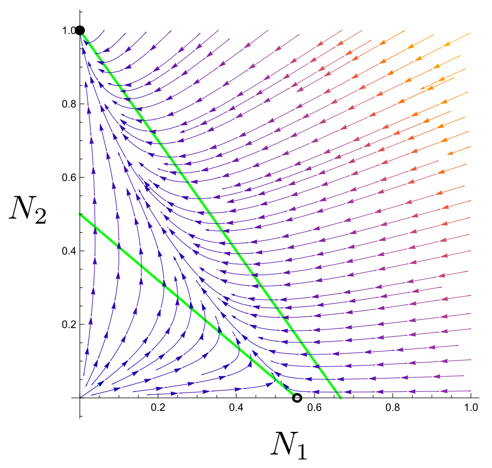

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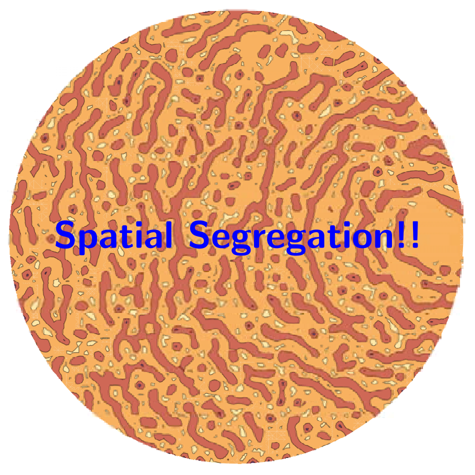

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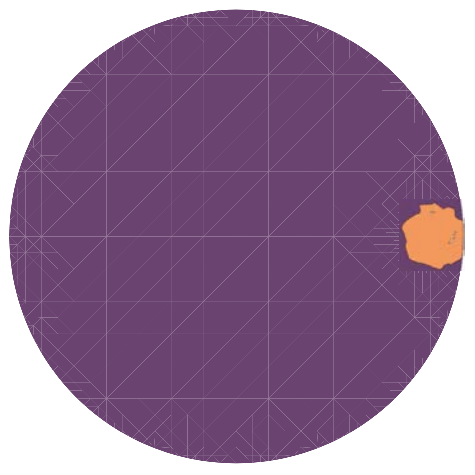

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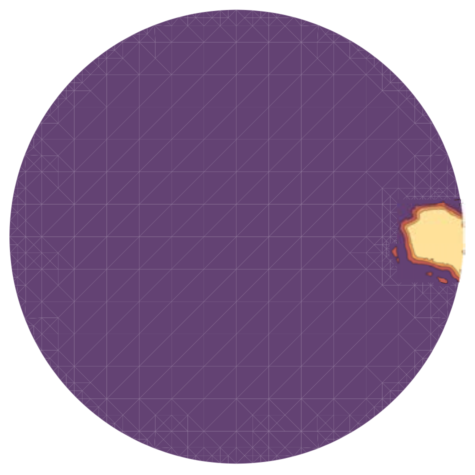

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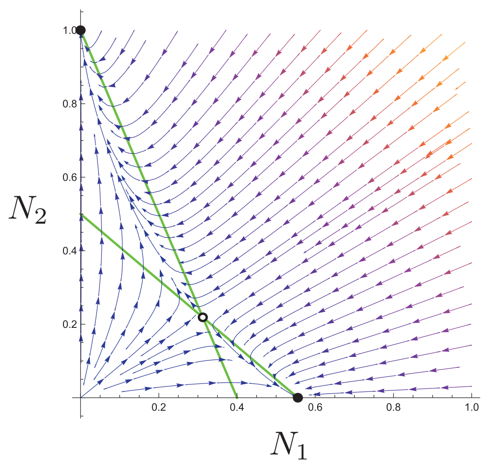

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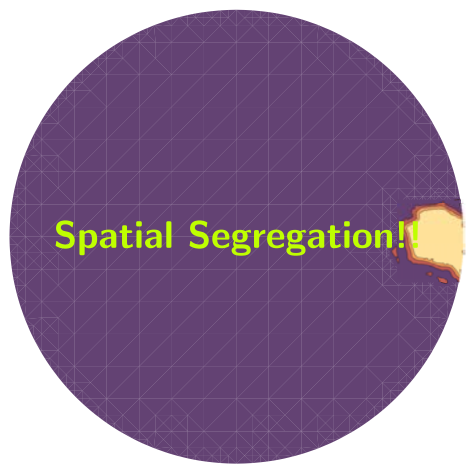

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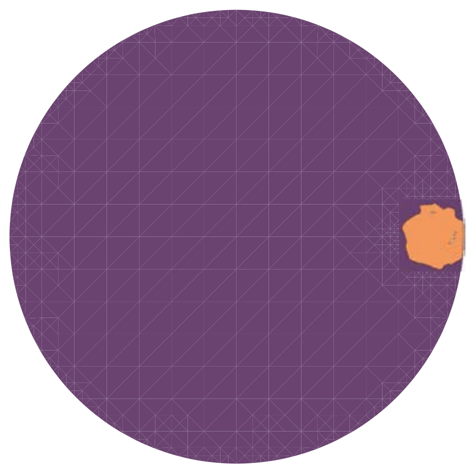

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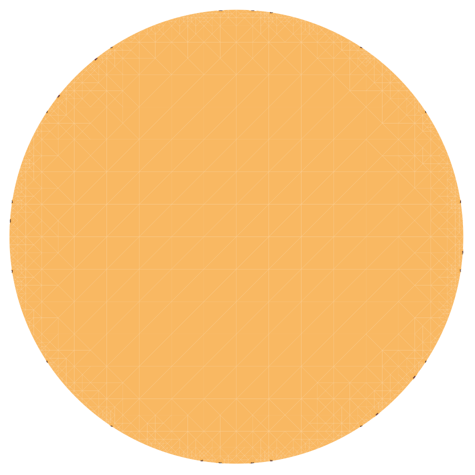

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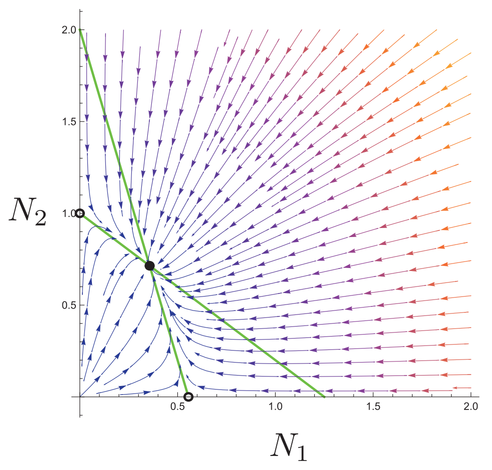

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Prologue

Mobility could be an important factor for the coexistence of competing species.

cf. Shigesada N, Kawasaki K, Teramoto E (1979) Spatial segregation of interacting species. *Journal of Theoretical Biology* **79**(1): 83–99.

Mimura M, Kawasaki K (1980) Spatial segregation in competitive interaction-diffusion equations. *Journal of Mathematical Biology* **9**(1): 49–64.

Mimura M, Ei SI, Ikota R (1999) Segregating partition problem in competition-diffusion systems. *Interfaces and Free Boundaries* **1**(1): 57–80.

Invasion of competitive alien species

“coexistence” $\Rightarrow$ invasion success

Invasion of competitive alien species

$$\text{“coexistence”} \quad \Rightarrow \quad \text{invasion success}$$

$$\text{“exclusion” of native species} \quad \Rightarrow \begin{cases} \text{invasion success} \\ \text{species exchange} \end{cases}$$

Prologue

Invasion of competitive alien species

"coexistence"	$\Rightarrow$	invasion success
"exclusion" of native species	$\Rightarrow$	invasion success species exchange
"exclusion" of alien species	$\Rightarrow$	invasion failure resistence of native system

Prologue

Invasion of competitive alien species

Invasion success of alien species $\Rrightarrow$ **Threat to native ecosystem**

Prologue

Invasion of competitive alien species

Invasion success of alien species $\Rrightarrow$ Change in ecosystem services

Prologue

Newly observed fish at Ishinomaki Fish Market

新顔の魚たち–石巻魚市場

新顔の魚たち

ウッカリカサゴ 2020/2/14 Sebastiscus tertius

カゴマトウダイ 2020/4/2 Cyttopsis rosea

テングダイ2020/9/17 Evistias acutirostris

オニテングハギ2020/9/28 Naso brachycentron

ギンザメ2020/10/14 Chimaera phantasma

ツバメウオ2020/10/14 Platax teira

出典:一般社団法人漁業情報サービスセンター 東北出張所
「変動する三陸～仙台湾の魚たち」 2020.12.23
https://www.jafic.or.jp/information/2020/12/23/509/

What is “competition”?

Outline

Prologue

What is “competition”?

A distinctive topic: Different stage-specific alien predator

Epilogue

What is “competition”?

What is “competition”?

Inductive definition of “competitive relation” in population dynamics

Relation between two populations with some **negative density effects** on the growth of the other population size.

$\leadsto$ Influence-based

What is “competition”?

Inductive definition of “competitive relation” in population dynamics

Relation between two populations with some negative density effects on the growth of the other population size.

Negative density effect on population growth

- Decrease in reproductivity
- Increase in death rate
- Decrease in survivability

What is "competition"?

Inductive definition of "competitive relation" in population dynamics

Relation between two populations with some **negative density effects** on the growth of the other population size.

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What is “competition”?

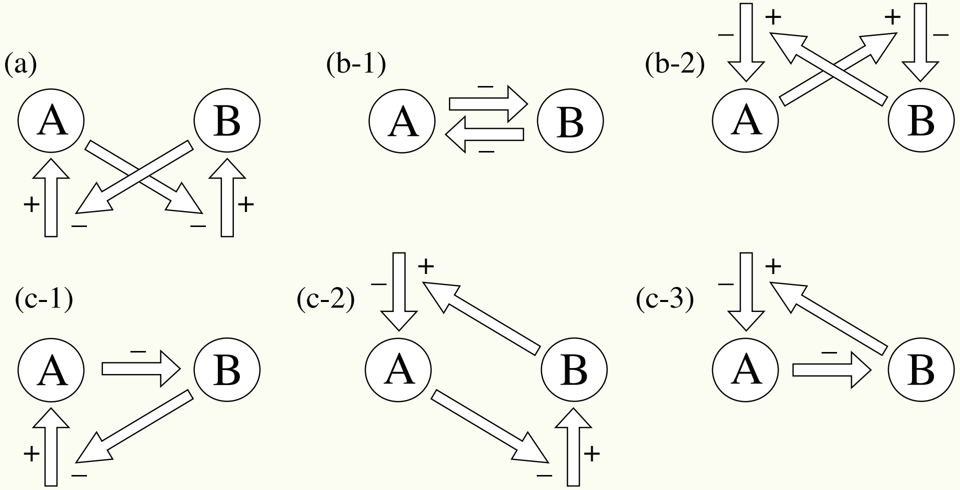
Inductive definition of “competitive relation” in population dynamics

Relation between two populations with some negative density effects on the growth of the other population size.

What reaction induces such a negative density effect on a population?

What is “competition”?

Causality-based classification of “competitive relation”



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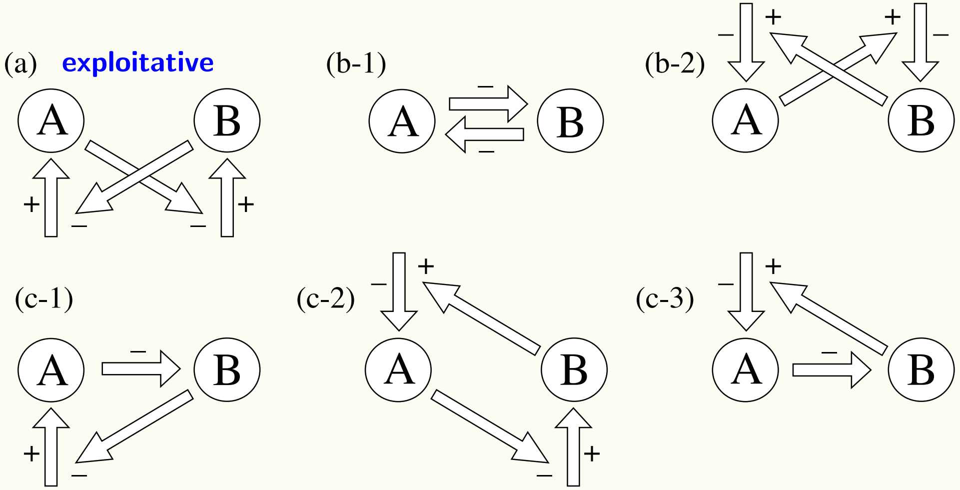

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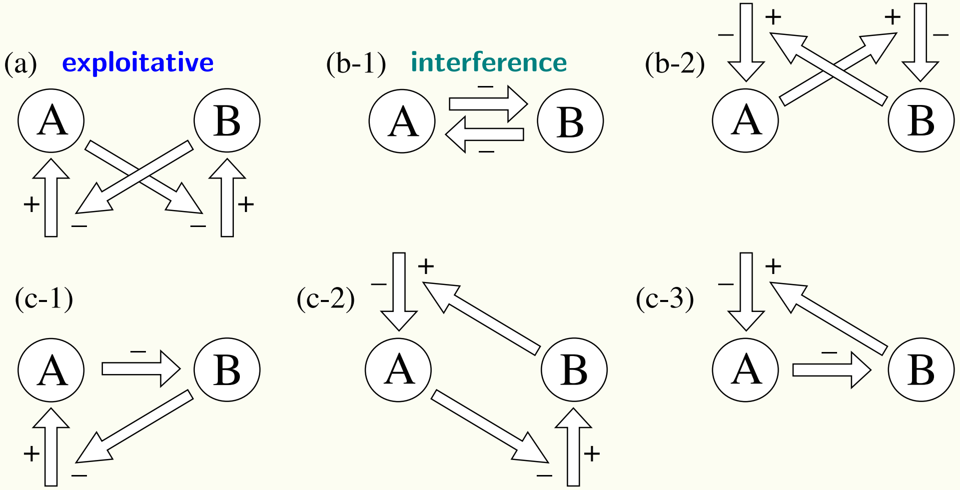

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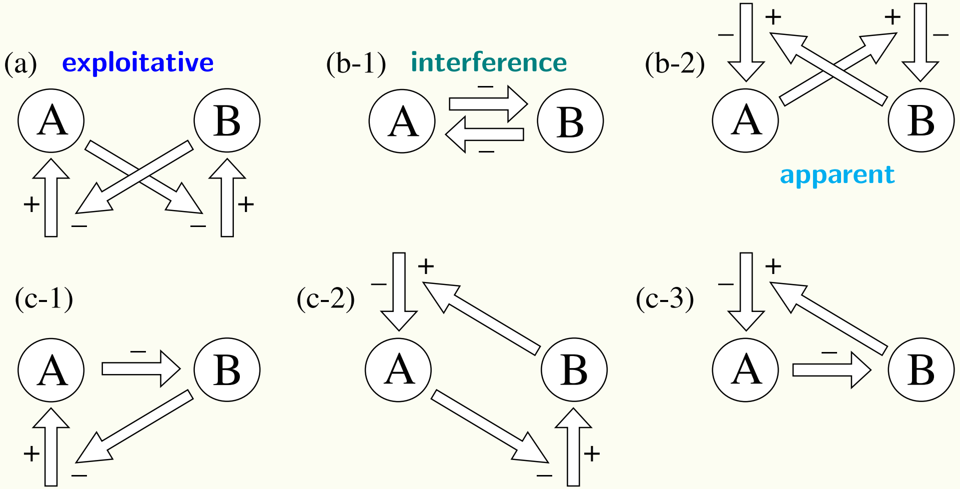

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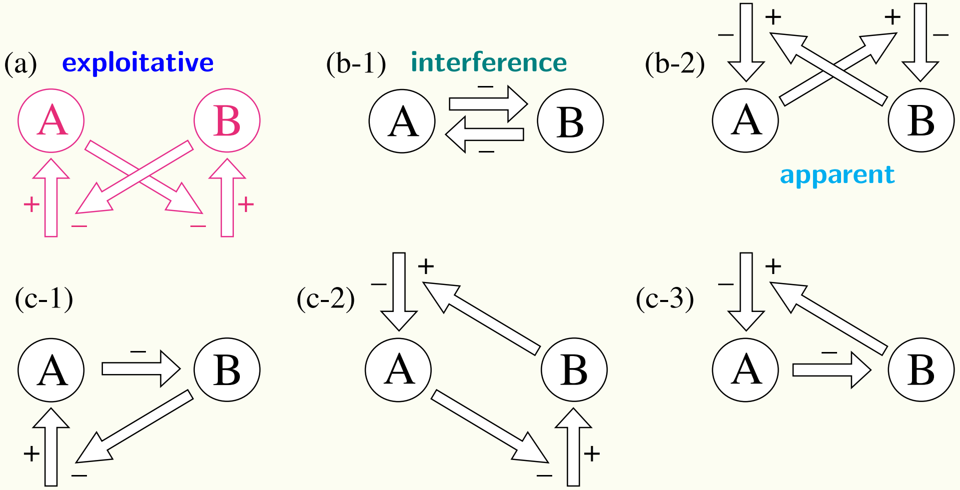
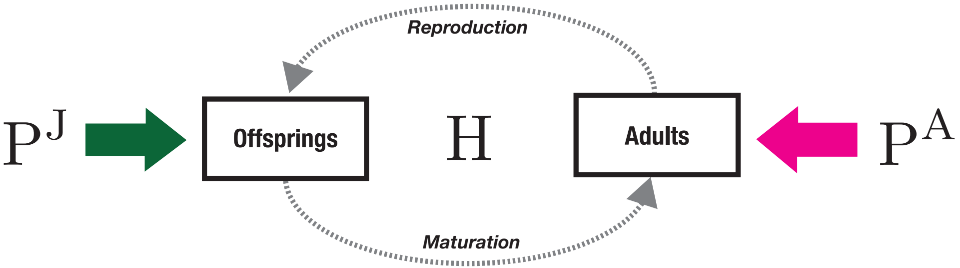


A distinctive topic:

Different stage-specific alien predator

A distinctive topic: Different stage-specific alien predator

Two predators with different stage-specific predation for a common prey



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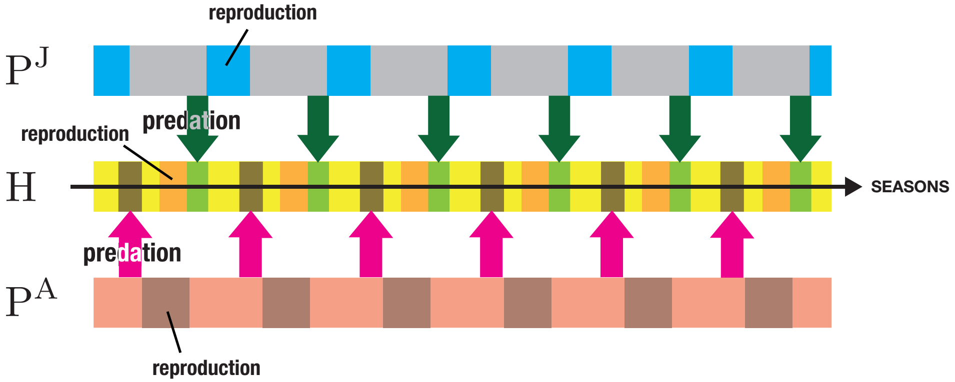


A distinctive topic: Different stage-specific alien predator

Prey population dynamics

$$H_{n+1} = F(H_n)H_n$$

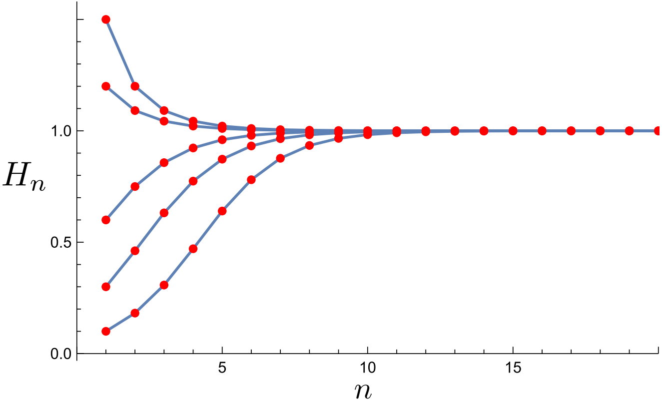
with the per capita reproduction function

$$F(H) = \frac{r_0}{1 + H/\beta} \quad (r_0 > 1)$$

cf. Beverton-Holt model

A distinctive topic: Different stage-specific alien predator

Prey population dynamics



cf. Beverton-Holt model ⇔ logistic equation model

A distinctive topic: Different stage-specific alien predator

Predator population dynamics

$$P_{n+1} = G(P_n, \mathcal{Z}_n)P_n$$

with

$$G(P_n, \mathcal{Z}_n) = \rho \frac{\mathcal{Z}_n}{P_n},$$

where $\mathcal{Z}_n$ is the total amount of preys predated by the population of predator P at the n th predation season.

A distinctive topic: Different stage-specific alien predator

Predator population dynamics

$$P_{n+1} = G(P_n, \mathcal{Z}_n)P_n \;= \rho\mathcal{Z}_n$$

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A distinctive topic: Different stage-specific alien predator

Two distinct models for prey-predator population dynamics

	Predator P^J	Predator P^A
Model J	native	alien
Model A	alien	native

A distinctive topic: Different stage-specific alien predator

Models for native prey-predator system

Model J
$$\begin{cases} H_{n+1} &= \Pi_{\mathrm{J}}(P_n^{\mathrm{J}})F(H_n)H_n \\ P_{n+1}^{\mathrm{J}} &= \rho_{\mathrm{J}}\{1-\Pi_{\mathrm{J}}(P_n^{\mathrm{J}})\}F(H_n)H_n \end{cases}$$

Model A
$$\begin{cases} H_{n+1} &= F\big(\Pi_{\mathrm{A}}(P_n^{\mathrm{A}})H_n\big)\Pi_{\mathrm{A}}(P_n^{\mathrm{A}})H_n \\ P_{n+1}^{\mathrm{A}} &= \rho_{\mathrm{A}}\{1-\Pi_{\mathrm{A}}(P_n^{\mathrm{A}})\}H_n \end{cases}$$

with the probability of successful escape from the predation, $\Pi_{\bullet}(P) = \mathrm{e}^{-a_{\bullet}P}$.

cf. Nicholson-Bailey model

Models for native prey-predator system

Model J

$$\begin{cases} h_{n+1} = e^{-p_n^{\mathrm{J}}} \dfrac{r_0}{1+h_n} h_n \\ p_{n+1}^{\mathrm{J}} = \alpha_{\mathrm{J}} \left(1 - e^{-p_n^{\mathrm{J}}}\right) \dfrac{r_0}{1+h_n} h_n \end{cases}$$

Model A

$$\begin{cases} h_{n+1} = \dfrac{r_0}{1+e^{-p_n^{\mathrm{A}}} h_n} e^{-p_n^{\mathrm{A}}} h_n \\ p_{n+1}^{\mathrm{A}} = \alpha_{\mathrm{A}} \left(1 - e^{-p_n^{\mathrm{A}}}\right) h_n \end{cases}$$

with non-dimensionalizing transformations.

A distinctive topic: Different stage-specific alien predator

Models for prey-predator population dynamics with invading alien predator

$$
\begin{cases}
H_{n+1} &= \Pi_{\mathrm{J}}(P_n^{\mathrm{J}})F\big(\Pi_{\mathrm{A}}(P_n^{\mathrm{A}})H_n\big)\Pi_{\mathrm{A}}(P_n^{\mathrm{A}})H_n \\
P_{n+1}^{\mathrm{J}} &= \rho_{\mathrm{J}}\{1-\Pi_{\mathrm{J}}(P_n^{\mathrm{J}})\}F\big(\Pi_{\mathrm{A}}(P_n^{\mathrm{A}})H_n\big)\Pi_{\mathrm{A}}(P_n^{\mathrm{A}})H_n \\
P_{n+1}^{\mathrm{A}} &= \rho_{\mathrm{A}}\{1-\Pi_{\mathrm{A}}(P_n^{\mathrm{A}})\}H_n
\end{cases}
$$

A distinctive topic: Different stage-specific alien predator

Models for prey-predator population dynamics with invading alien predator

$$
\begin{cases}
h_{n+1} = e^{-p_n^{J}} \dfrac{r_0}{1+e^{-p_n^{A}} h_n} e^{-p_n^{A}} h_n \\
p_{n+1}^{J} = \alpha_{J} \left(1 - e^{-p_n^{J}}\right) \dfrac{r_0}{1+e^{-p_n^{A}} h_n} e^{-p_n^{A}} h_n \\
p_{n+1}^{A} = \alpha_{A} \left(1 - e^{-p_n^{A}}\right) h_n
\end{cases}
$$

with non-dimensionalizing transformations.

A distinctive topic: Different stage-specific alien predator

Models for native prey-predator system

Model J

$$\begin{cases} h_{n+1} &= \mathrm{e}^{-p_n^{\mathrm{J}}} \dfrac{r_0}{1+h_n} h_n \\ p_{n+1}^{\mathrm{J}} &= \alpha_{\mathrm{J}} \left(1-\mathrm{e}^{-p_n^{\mathrm{J}}}\right) \dfrac{r_0}{1+h_n} h_n \end{cases}$$

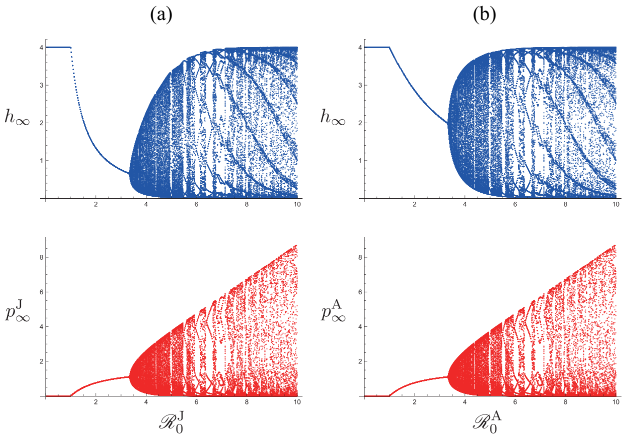
Model A

$$\begin{cases} h_{n+1} &= \dfrac{r_0}{1+\mathrm{e}^{-p_n^{\mathrm{A}}} h_n} \mathrm{e}^{-p_n^{\mathrm{A}}} h_n \\ p_{n+1}^{\mathrm{A}} &= \alpha_{\mathrm{A}} \left(1-\mathrm{e}^{-p_n^{\mathrm{A}}}\right) h_n \end{cases}$$

with non-dimiensionalizing transformations.

A distinctive topic: Different stage-specific alien predator

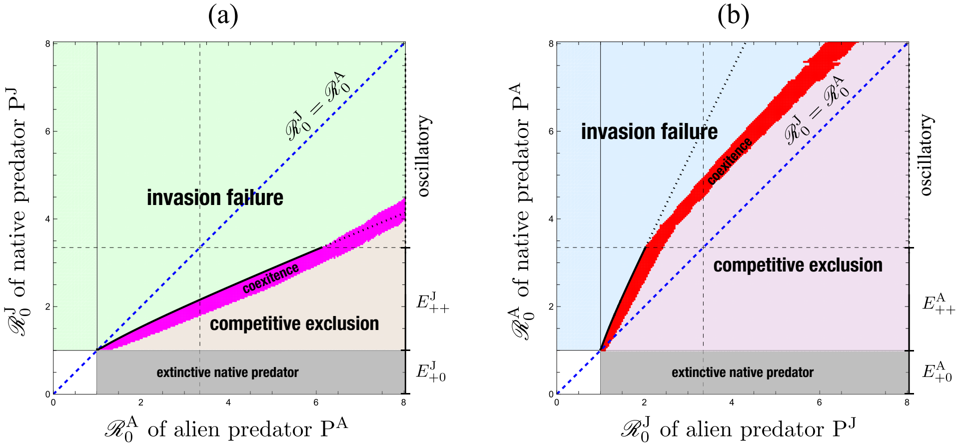
Bifurcation diagram for native prey-predator system



(a) Model J; (b) Model A, where $\mathscr{R}_0^{\bullet} := \rho_{\bullet} a_{\bullet} \beta(r_0 - 1)$ is the basic predator replacement number for predator $\mathrm{P}^{\bullet}$.

A distinctive topic: Different stage-specific alien predator

Final state after the invasion of alien predator, (a) Model J; (b) Model A



A distinctive topic: Different stage-specific alien predator

- **The juvenile-specific predator has the invadability higher than the adult-specific predator.**

A distinctive topic: Different stage-specific alien predator

- **The juvenile-specific predator has the invadability higher than the adult-specific predator.**
- **The prey-predator system with the juvenile-specific predator would be more resistant to the invasion of alien predator.**

A distinctive topic: Different stage-specific alien predator

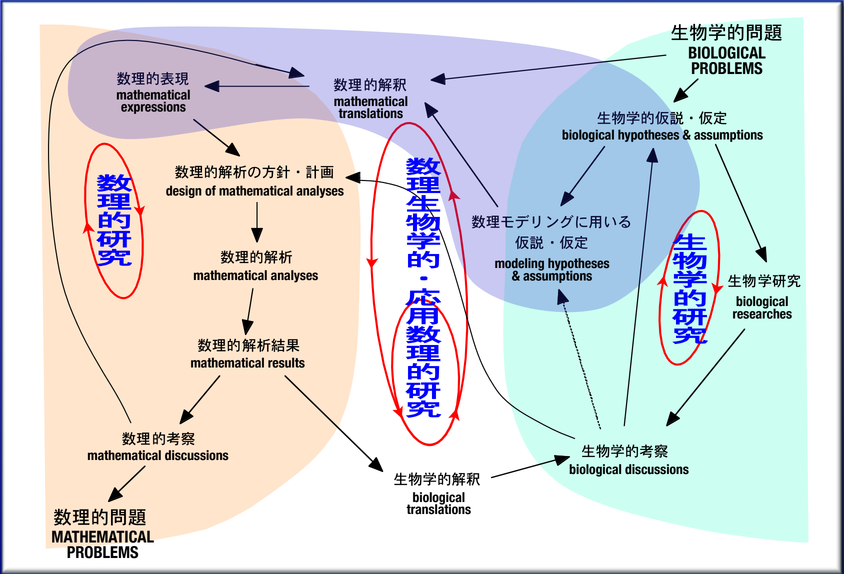
- The juvenile-specific predator has the invadability higher than the adult-specific predator.
- The prey-predator system with the juvenile-specific predator would be more resistant to the invasion of alien predator.
- The prey-predator system with the adult-specific predator would be more vulnerable to the invasion of alien predator.

A distinctive topic: Different stage-specific alien predator

- The juvenile-specific predator has the invadability higher than the adult-specific predator.
- The prey-predator system with the juvenile-specific predator would be more resistant to the invasion of alien predator.
- The prey-predator system with the adult-specific predator would be more vulnerable to the invasion of alien predator.
- The persistent prey-predator system would be composed of the juvenile-specific predator.

Epilogue

Epilogue



生物学的問題
BIOLOGICAL PROBLEMS
数理的表現
mathematical expressions
数理的解釈
mathematical translations
生物学的仮説・仮定
biological hypotheses & assumptions
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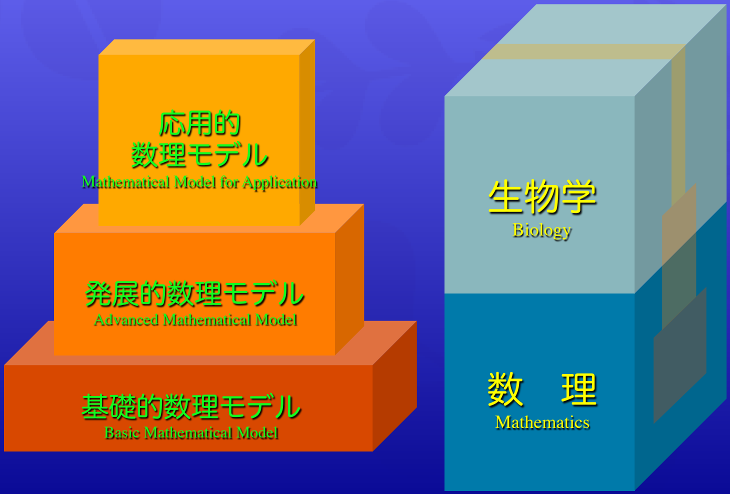
数理的問題
MATHEMATICAL PROBLEMS

Epilogue

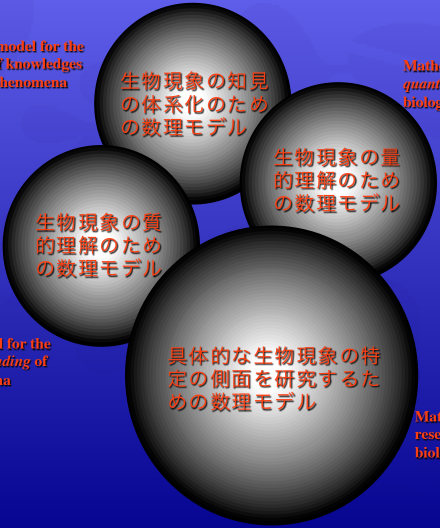


応用的
数理モデル
Mathematical Model for Application
発展的数理モデル
Advanced Mathematical Model
基礎的数理モデル
Basic Mathematical Model
生物学
Biology
数　理
Mathematics

Epilogue

Mathematical model for the *systemization* of knowledges for biological phenomena

Mathematical model for the *quantitative understanding* of biological phenomena



Mathematical model for the *qualitative understanding* of biological phenomena

Mathematical model for the research on a *specific* biological phenomena

Epilogue

説　明
explanation

実　験
experiment

記　述
description

体 系 化
systemization

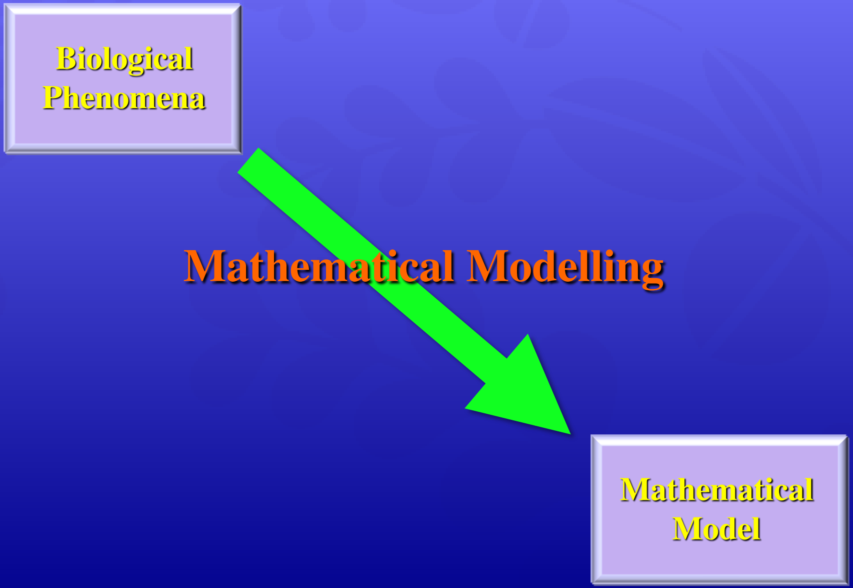
予　測
prediction

モデル開発
model development

理　解
understanding

数理的興味
mathematical interest

Epilogue


Biological Phenomena
Mathematical Modelling
Mathematical Model

Epilogue

Population Dynamics

is the nature of the spatio-temporal variation of biological population size (i.e. density etc.).

繁殖 reproduction
闘争 fighting
競争 competition
共生 mutualism
捕食 predation
寄生 parasitism
etc.

Mathematical Modelling

Mathematical Model

Epilogue

Population Dynamics

is the nature of the spatio-temporal variation of biological population size (i.e. density etc.).

Density Effect

繁殖 reproduction
闘争 fighting
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共生 mutualism
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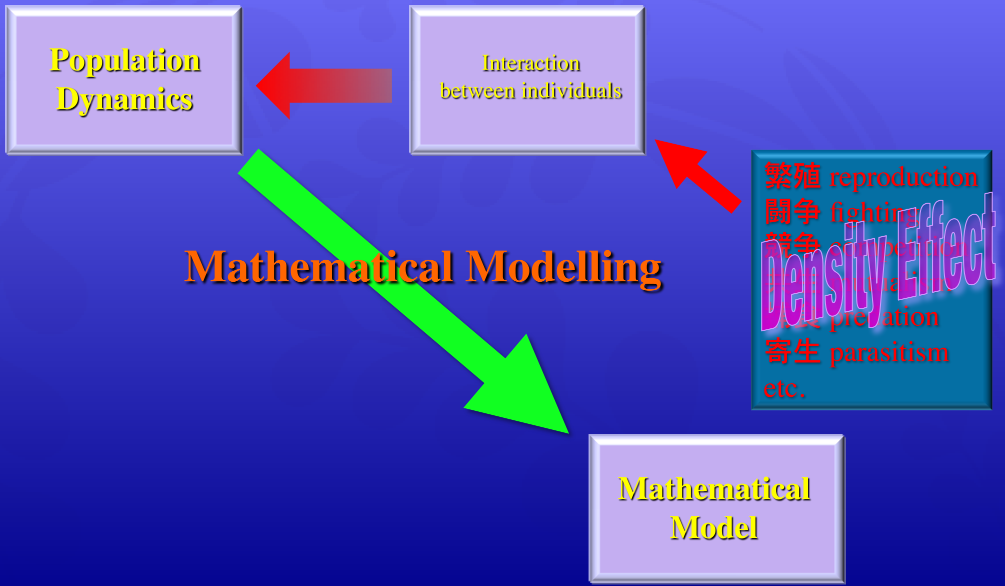
Mathematical Modelling

Mathematical Model

Epilogue


Population Dynamics
Interaction between individuals
繁殖 reproduction
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mutualism
predation
寄生 parasitism
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Density Effect
Mathematical Modelling
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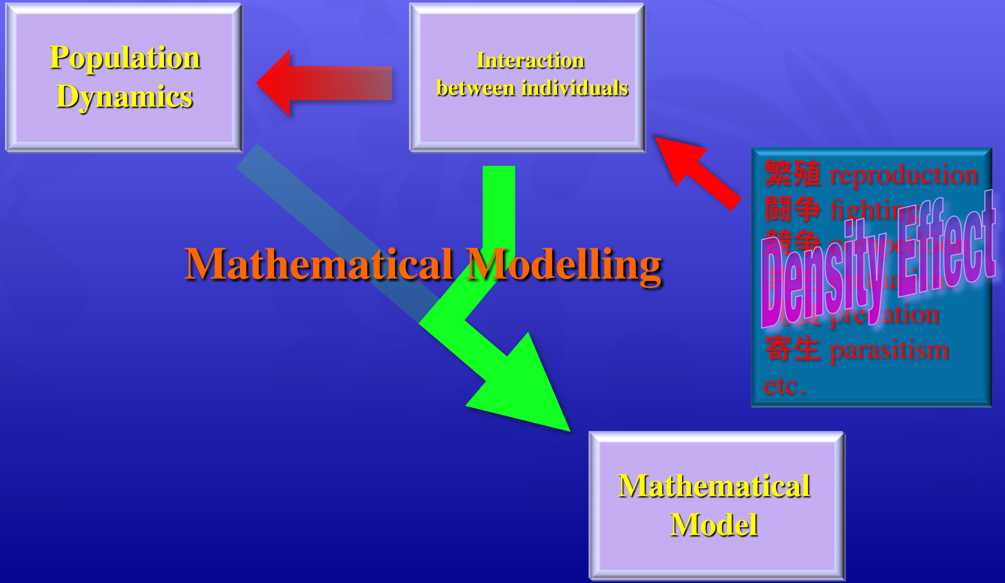
Epilogue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繁殖 reproduction
闘争 fighting
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predation
寄生 parasitism
etc.
Density Effect

Epilogue

What mathematical model is reasonable from the biological viewpoint?

What mathematical structure is appropriate for the reasonable modeling?

[L-1] $$\frac{dN(t)}{dt} = \{r_0 - \beta N(t)\}N(t)$$

[L-2] $$\frac{dN(t)}{dt} = r_0\left\{1 - \frac{N(t)}{K}\right\}N(t)$$

[L-3] $$\frac{dN(t)}{dt} = r_0 N(t) - b\{N(t)\}^2$$

[L-4] $$\frac{dN(t)}{dt} = \{r_0 - \beta N(t)\}N(t) - b\{N(t)\}^2$$

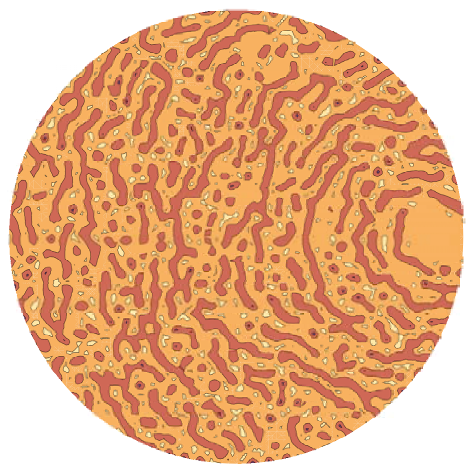
r_0: intrinsic growth rate

Epilogue

Reasonability of modeling depends on

i) purpose of theoretical research;
ii) available data/knowledge/hypothesis;
iii) design of mathematical analysis.

Epilogue

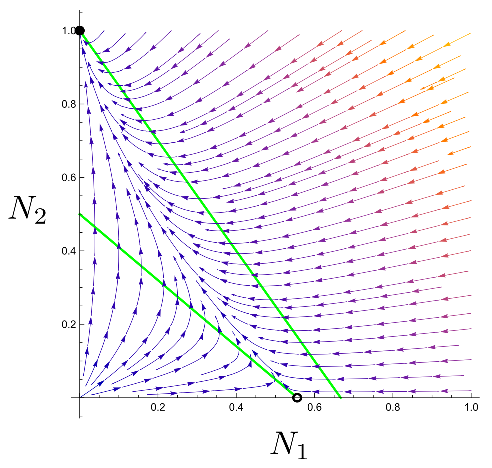


Lotka-Volterra competition system
with diffusion

$$\begin{cases} \dfrac{\partial N_1}{\partial t} = D_1 \nabla^2 N_1 + (r_1 - \beta_1 N_1 - \gamma_{12} N_2) N_1 \\ \dfrac{\partial N_2}{\partial t} = D_2 \nabla^2 N_2 + (r_2 - \beta_2 N_2 - \gamma_{21} N_1) N_2 \end{cases}$$

$D_1 = 2.0 \times 10^{-5}$; $r_1 = 1.0$; $\beta_1 = 1.8$; $\gamma_{12} = 2.0$;
$D_2 = 2.0 \times 10^{-6}$; $r_2 = 1.0$; $\beta_2 = 1.0$; $\gamma_{21} = 1.5$

Epilogue



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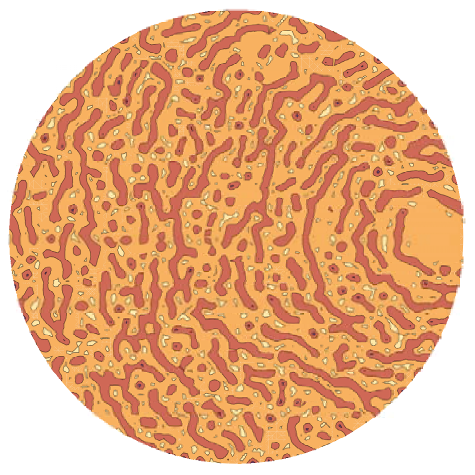
Epilogue



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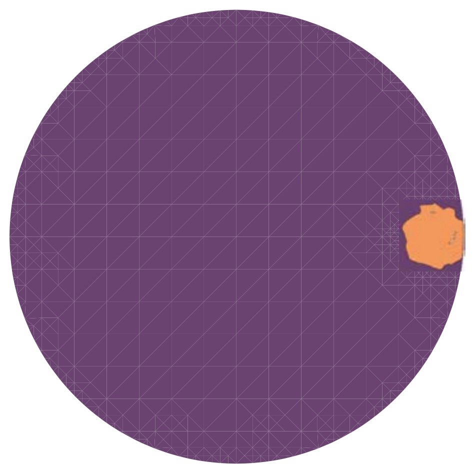

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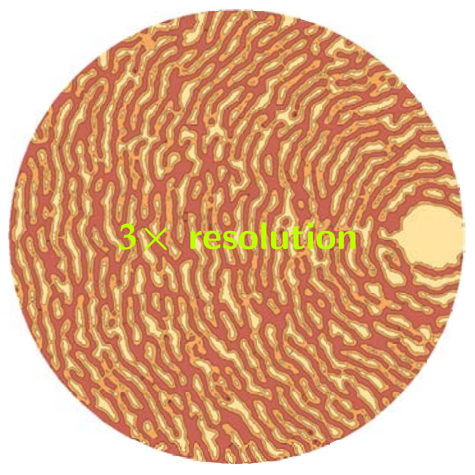
Epilogue



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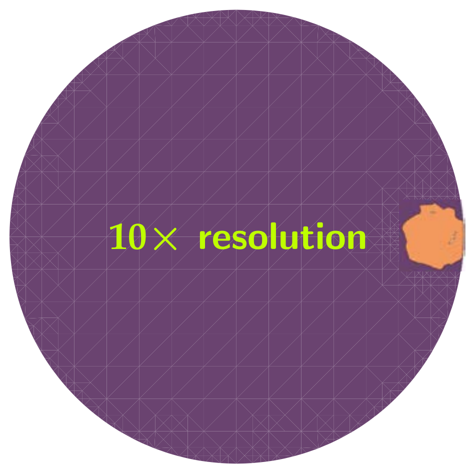
Epilogue



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Epilogue

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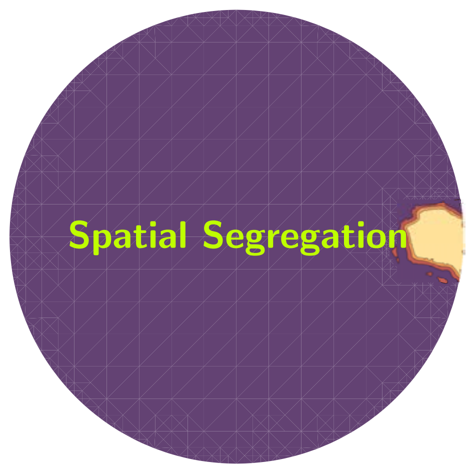
Epilogue



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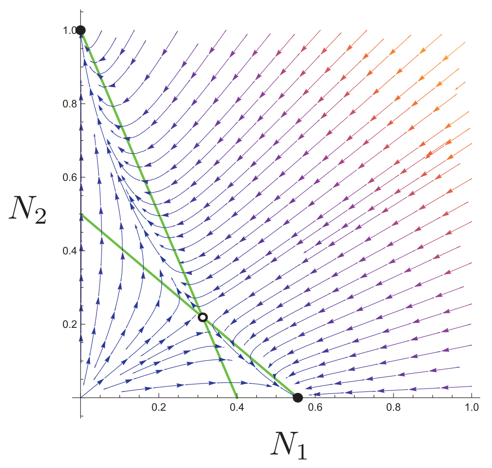
Epilogue



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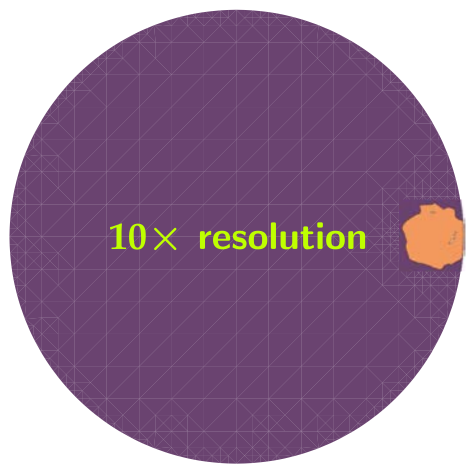
Epilogue



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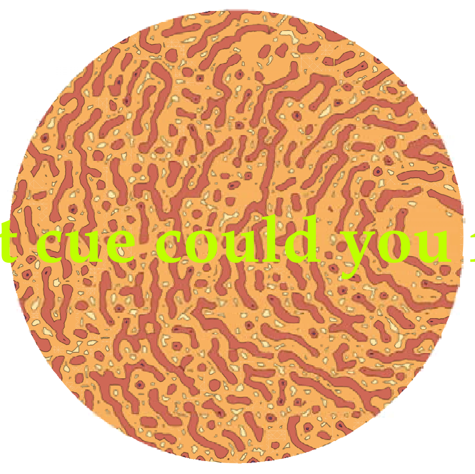
A trite lesson

Not be tricked by numerics!

A sophisticated lesson

Adventitious numerics could provide cues for new scientific idea.



What cue could you find?

Thank you for your attention!