Abstract Collection

Schatten p-class for holomorphic composition operator on smoothly bounded strongly pseudoconvex domain

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Based on my recent complex analysis paper joint with W. Yang and C. Yuan, I will present such a full characterization of the Schatten p-class for a holomorphic composition operator on the smoothly bounded strongly pseudoconvex domain that if

$$\begin{cases} \tau = 0 & \text{as } p \in \left(\frac{2n}{n+1}, \infty\right); \\ \tau > \left(\frac{n+1}{2p}\right)\left(\frac{2n}{n+1} - p\right) & \text{as } p \in \left(0, \frac{2n}{n+1}\right], \end{cases}$$

then

$$cp \in scp\big(A^2(\Omega)\big) \Longleftrightarrow \left(\int_\Omega \left(\delta(z)^{n+1+2\tau} \int_\Omega \big|K_\tau\big(z, p(w)\big)\big|^2 rdv(w)\right)^{\frac{p}{2}} K(z,z) rdv(z)\right)^{\frac{1}{p}} < \infty$$

thereby improving Theorem 1.1 in S.-Y. Li's 1995 Amer. J. Math. paper "Trace ideal criteria for composition operators on Bergman spaces" from $p > \frac{2n}{n+1}$ (known case) to $p \leq \frac{2n}{n+1}$ (open case).

Intrinsic Ultracontractivity for planar domains with wide access property

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Keywords: Intrinsic ultracontractivity, wide access, planar domain, capacitary width
2020 MSC: Primary 35K08H10

For a domain D in the Euclidean space we denote by $p_D(t, x, y)$, $t > 0$, $x, y \in D$, the Dirichlet heat kernel for D, i.e., the fundamental solution to the heat equation subject to the Dirichlet boundary condition $u(t, x) = 0$ for $x \in \partial D$ and $t > 0$ and $u(0, x) = f(x)$ for $x \in D$. Davies and Simon [3] introduced the notion of IU (intrinsic ultracontractivity), i.e., the Dirichlet Laplacian has no essential spectrum and the heat kernel enjoys

$$c_t\varphi(x)\varphi(y) \leq p_D(t, x, y) \leq C_t\varphi(x)\varphi(y) \quad \text{for all } x, y \in D$$

with the ground state $\varphi(x)$. IU may be regarded as a parabolic boundary Harnack principle; it implies various interesting properties such as the Cranston-McConnell inequality. IU holds for very complicated domains (e.g. Davis [2]). Bañuelos and Davis[1] gave a simple geometric characterization of IU for a planar domain above the graph of a function, It is challenging to find a geometric characterization for IU for more general domains. In this talk we deal with simply connected planar domains with wide access property and unveil properties toward geometric characterization for IU.

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Automorphism groups of closed Klein surfaces with extremal disks

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2020 MSC: Primary 30F50; Secondary 30F10.

A surface with a dianalytic structure is called a Klein surface. A (non-orientable) closed Klein surface of genus $g \geq 3$ is called extremal surface if a disk of radius r_g in the hyperbolic plane is isometrically embedded in the surface, where r_g is the largest radius determined by g. We know that the largest genus for which an extremal Klein surface can contain more than one such disk is 6. In this talk we consider the group of automorphisms of Klein surfaces, in particular those of $g = 6$.

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Progress on the Shapiro-Sundberg problems

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2020 MSC: Primary 47B33; Secondary 30H10, 30H20

In 1990 Shapiro and Sundberg raised two problems concerning composition operators acting on the Hardy space. One (the path component problem) is to characterize path connection between two composition operators and the other (the compact difference problem) is to characterize compact difference of composition operators. Those Shapiro-Sundberg Problems have been studied by many experts for the past three decades. Quite recently, we have solved the compact difference problem in three different ways: the joint Carleson condition, the reproducing kernel thesis and the modified Carleson condition. However, the path component problem is still open. In this talk I will present background, motivation, progress, our contributions and further problems related to the Shapiro-Sundberg problems. This presentation is based on recent joint works with Koeun Choi, Hyungwoon Koo, Inyoung Park and Jongho Yang.

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A fiber space over the TeichmÏler space of circle diffeomorphisms in the Zygmund class

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The groups of circle diffeomorphisms of certain regularity have been studied in the framework of the theory of the universal Teichmüller space. In previous papers [1] and [4], we considered the class of orientation-preserving circle diffeomorphisms whose derivatives are γ-Hölder continuous for $0 < \gamma < 1$, and laid the foundation for this Teichmüller space. In the limiting case of this γ-Hölder continuity as $\gamma \to 1$, one can adopt the Zygmund continuous condition. The corresponding Teichmüller space T^Z of those circle diffeomorphisms has been defined by Tang and Wu [3]. They proved that the Schwarzian derivative map $\mu \mapsto S_{f_\mu}$ and the pre-Schwarzian derivative map $\mu \mapsto \log (f_\mu)'$ from the space M^Z of Beltrami coefficients μ of linear decay order towards the boundary to the corresponding complex Banach spaces A^Z and B^Z of holomorphic functions are holomorphic. Here, f_μ stands for a conformal homeomorphism of the unit disk quasiconformally extendable to its exterior with complex dilatation μ and normalization $f_\mu(\infty) = \infty$. In [2], we proved that the Schwarzian derivative map $S : M^Z \to A^Z$ is a holomorphic split submersion. It follows from this fact that the Bers embedding $\alpha : T^Z \to A^Z$ defined by factorizing S by the Teichmüller projection $M^Z \to T^Z$ is a homeomorphism onto its image, and hence T^Z is equipped with the complex Banach manifold structure of

A^2. In this talk, we consider the fiber space $\tilde{\mathcal{T}}^Z$ over $\alpha\left(T^Z\right) \cong T^Z$ that is the image of the pre-Schwarzian derivative map $L : \mu \mapsto \log\left(f_\mu\right)'$ in B^Z. For $\phi \in B^Z, \Lambda(\phi) = \phi'' - \left(\phi'\right)^2/2$ belongs to A^Z. Then, Λ satisfies $S = \Lambda \circ L$ on M^Z, and Λ maps $\tilde{\mathcal{T}}^Z$ onto $\alpha\left(T^Z\right)$. Concerning this map Λ, we obtain the following result. Theorem. $\Lambda : \tilde{\mathcal{T}}^Z \to \alpha\left(T^Z\right)$ is a holomorphic split submersion, and in fact, $\tilde{\mathcal{T}}^Z$ is a disk bundle over the Teichmuller space T^Z with the projection Λ. The proof has been announced in arXiv:2311.15521.

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Mappings of finite directional dilatations and boundary correspondence in $\mathbb{R}^n$

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Keywords: Teichmüller ring, modulus of a ring, boundary behavior, directional dilatation
2020 MSC: Primary 30C65; Secondary 26B35, 30C75, 31B15.

In our talk, we investigate the boundary correspondence problems for mappings with weaken regularity assumptions, i.e., from Sobolev class $W^{1,n-1}$. The main results rely on the multidimensional Teichmüller theorem on separating rings recently established in [GSV]. A Lipschitz and weak Hölder type continuity on the boundary under appropriate integral conditions involving directional dilatations will be discussed. Some needed background will be presented as well. This is joint work with Toshiyuki Sugawa and Matti Vuorinen.

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Brody curves on Fermat surface of degree six

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2020 MSC: Primary 30D05; Secondary 32Q45.

A holomorphic map from the complex line to the n-dimensional complex projective space is called a Brody curve if its spherical derivative is bounded. In 2010, Eremenko applied potential theory to study Brody curves omitting n hyperplanes in general position and showed that these curves have growth order at most one, normal type. In this talk, we will characterize Brody curves on the degree six Fermat surface in the three dimensional complex projective space based on Eremenko's potential theoretical method.
This is a joint work with Sai Kee Yeung.

On coefficients of inverse of convex functions

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2020 MSC: Primary 30C50; Secondary 30C45

The main subject of interest of this talk is the problem of estimating $|A_9|$, which is the modulus of the ninth coefficient of the inverse of a convex function belonging to the class $\mathcal{K}$. It was shown almost 50 years ago that $|A_n|$, where $n \geq 10$, can exceed 1. On the other hand, it is known that $|A_n| \leq 1$ for n ranging from 2 to 8. Until now, the problem of finding a sharp bound of $|A_9|$ has been unsolved. We present a new approach to solving it. Some related problems are also formulated.

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Extended generalized complete elliptic integrals and related bounding inequalities

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Keywords: Complete elliptic integrals; extended beta function; extended Gauss's hypergeometric function; Mellin transform; Laguerre polynomials; Log-convexity; Turan-type inequality.
2020 MSC: Primary 33C65, 33C75, 33E05; Secondary 26A33, 78A40,78A45.

We introduce extended generalized complete elliptic integrals of the first and the second kind by making use of extended Gauss's hypergeometric function, for which the usual properties and representations are extended in a simple manner. Log-convexity property and Turan-type inequalities are proved for these generalized elliptic integrals. In addition, we deduce several special values and provide connections with certain higher transcendental functions as new representations for special parameters. Functional bounds, Mellin transforms, certain infinite series representations containing Laguerre polynomials, and numerous differentiation formulas are also deduced.

Characterizations of circle homeomorphisms of different regularities in the universal Teichmüller space

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2020 MSC: Primary 30F60; Secondary 30C40, 30C62, 30E25.

Given in this talk are a summary of results on characterizations of circle homeomorphisms of different regularities (quasisymmetric, symmetric, or $C^{1+\alpha}$) in terms of Beurling-Ahlfors extension, Douady-Earle extension, and Thurston's earthquake representation of an orientation-preserving circle homeomorphism, and a brief account of corresponding characterizations of the elements of the tangent spaces of these sub Teichmüller spaces at the base point in the universal Teichmüller space.

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A Bochner-type integral inequality for nonnegative holomorphic sectional curvature

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2020 MSC: 53C07, 58E15

By establishing a Bochner-type integral inequality for compact Kähler manifolds with HSC (nonnegative holomorphic sectional curvature), we prove that a compact Kähler manifold with quasi-positive HSC is rationally connected, thereby affirming a question posed by Yang and Matsumura and extending Yau's conjecture. We also demonstrate that a compact simply connected Kähler manifold with nonnegative HSC is rationally connected. Additionally, we show that a non-projective compact Kähler 3-dimensional manifold with nonnegative HSC is either a torus $\mathbb{T}^3$ or a $\mathbb{P}^1$-bundle over a torus $\mathbb{T}^2$. This talk is based on the joint work with Xi Zhang.

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Fekete-Szegïnequalities in higher dimensions

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Let $\mathbb{B}$ be the unit ball of a complex Banach space. First, we give the Fekete-Szegö inequality for all normalized starlike mappings on $\mathbb{B}$. Next, we will generalize the Fekete-Szegö inequality for normalized univalent functions on the unit disc to that for the first elements of g-Loewner chains on $\mathbb{B}$. This is joint work with Gabriela Kohr and Mirela Kohr.

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The precise form of Ahlfors' Second Fundamental Theorem of covering surfaces.

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2020 MSC: Primary 51M09; Secondary; 30D35, 30D45, 52B60.

A simply connected covering surface $\Sigma = \left(f, \overline{\Delta}\right)$ over the unit Riemann sphere S is an orientation-preserving, continuous, open and finite-to-one mapping f from the closed unit disk $\overline{\Delta}$ into the sphere S. Here open means that f can be extended continuous and open to a neighborhood of $\overline{\Delta}$. We denote by **F** all simply connected surfaces. Let $E_q = \{a_1, a_2, \ldots, a_q\}$ be a set on the unit Riemann sphere consisting of q distinct points with $q > 2$. Ahlfors second fundamental theorem (SFT) states that there exists a positive number h depending only on E_q, such that for any surface $\Sigma = \left(f, \overline{\Delta}\right) \in \mathbf{F}$,

$$(q-2)\, A(\Sigma) < 4\pi \overline{n}(\Sigma) + hL(\partial\Sigma),$$

where Δ is the unit disk, $A(\Sigma)$ is the spherical area of Σ, $L(\partial\Sigma)$ is the spherical length of the boundary curve $\partial\Sigma = (f, \partial\Delta)$, and $\overline{n}(\Sigma) = \# f^{-1}(E_q) \cap \Delta$. If we define $R(\Sigma) = R(\Sigma, E_q)$ to be the error term in Ahlfors' SFT, say,

$$R(\Sigma) = (q-2)\, A(\Sigma) - 4\pi \overline{n}(\Sigma),$$

then Ahlfors' SFT reads

$$H_0 = \sup_{\Sigma \in \mathbf{F}} \left\{ \frac{R(\Sigma)}{L(\partial\Delta)} : \Sigma = \left(f, \overline{\Delta}\right) \right\} < +\infty.$$

We call $H_0 = H_0(E_q)$ Ahlfors' constant for simply connected surfaces. In this talk, I will introduce my recent work which identify the precise bound $H_0 = H_0(E_q)$.

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Analytic adjoint ideal sheaves and residues of harmonic forms

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Given a complex manifold X and a reduced snc divisor D on it, there are residue functions, each of which holomorphically "deforms" an L^2 norm on X to an L^2 norm on specific strata of D. The analytic adjoint ideal sheaves are the algebraic manifestation of such residue functions, which is the key ingredient in our solution to the conjecture of Fujino on the injectivity property on compact Kähler lc pairs. In this talk, the residue functions and analytic adjoint ideal sheaves are introduced. Their use in facilitating the induction on the dimension of the strata of D rather than on the number of components of D will be explained. Furthermore, when X is compact Kähler, there is an intimate relation between a certain residue of harmonic forms (with values in some semi-positive line bundle) and the exact sequence of cohomology groups of the adjoint ideal sheaves. This will be illustrated with examples. This is joint work with Young-Jun Choi and Shin-ichi Matsumura.

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Riesz conjugate functions theorem for harmonic quasiconformal mappings

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2020 MSC: Primary 30H10; Secondary 30C62.

In this talk, I would like to introduce some recent results related to the Riesz conjugate functions theorem. Namely, we generalize Riesz conjugate functions theorem for planar harmonic K-quasiregular mappings (when $1 < p \leq 2$) and harmonic K-quasiconformal mappings (when $2 < p < \infty$) in the unit disk. Moreover, if $K = 1$, then our constant coincides with the classical analytic case. For n dimensional case ($n > 2$), we also obtain the Riesz conjugate functions theorem for invariant harmonic K-quasiregular mappings when $1 < p \leq 2$. This is joint work with Professor Liu Jinsong.

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Semigroup of composition operators on analytic function spaces

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I will talk some results on the semigroup of composition operators on analytic function spaces. In particular, we show that no non-trivial semigroup consisting of analytic self-maps of the unit disk generates a strongly continuous semigroup of composition operators on Q_p spaces for $p > 0$. For a semigroup, consisting of analytic self-maps of the unit disk, with inside Denjoy-Wolff point, we give some characterizations of the strongly continuous semigroup of composition operators on Q_p and Morrey spaces. This is joint work with Fanglei Wu and Fangmei Sun.

The L^1-L^∞-geometry on Teichmüller space

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2020 MSC: primary 32G05, 32G15, 53G60, 58A05, 58A30. Secondary 32U15, 57M50, 53B05, 32Q45, 32V20

The Teichmüller space $\mathcal{T}_g$ of closed Riemann surfaces of genus g is the deformation space of marked Riemann surfaces of genus g. The Teichmüller space $\mathcal{T}_g$ admits a canonical complex structure inherited from holomorphic families of quasiconformal mappings. The (holomorphic) tangent and cotangent spaces on $\mathcal{T}_g$ are described by infinitesimal deformations of quasiconformal mappings and the space of holomorphic quadratic differentials via the natural pairing (the Serre duality) between measurable L^∞ $(-1,1)$-forms and holomorphic L^1-quadratic differentials. The natural complex Finsler metric $\boldsymbol{\kappa}$ on the holomorphic tangent bundle $T\mathcal{T}_g$ on $\mathcal{T}_g$ defined from the pairing (duality) is called the Teichmüller metric. From this situation, we call the geometry of the Teichmüller space with the Teichmüller metric and the L^1-norm (cometric) of holomorphic quadratic differentials the *L^1-L^∞-geometry on the Teichmüller space*. In the L^1-L^∞-geometry, the *Teichmüller-Beltrami map*

$$\mathbf{tb}\colon \mathcal{Q}_g \ni q \mapsto \left[\|q\|\frac{\overline{q}}{|q|}\right] \in T\mathcal{T}_g$$

plays important and fundamental roles, where $\mathcal{Q}_g \to \mathcal{T}_g$ is the holomorphic vector bundle of holomorphic quadratic differentials and $\|\cdot\|$ is the L^1-norm. Though the Teichmüller-Beltrami map is a fiber-bundle isomorphism but not a vector-bundle isomorphism, the Teichmüller-Beltrami map gives a natural duality between the Teichmüller metric and the L^1-norm functions on $\mathcal{Q}_g$:

$$\boldsymbol{\kappa}(\mathbf{tb}(q)) = \|q\| \quad (q \in \mathcal{Q}_g).$$

In this talk, we will deal with the following two folklore:

- the Teichmüller-Beltrami map is a real analytic diffeomorphism on each stratum of $\mathcal{Q}_g$;
- the Teichmüller metric is real analytic on the image of each stratum,

where the space $\mathcal{Q}_g$ is stratified in terms of the structures of zeros of holomorphic quadratic differentials. We also discuss a new duality between the Teichmüller metric and the L^1-norm functions on $\mathcal{Q}_g$.

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Propagation of waves on non-homogeneous strings

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2020 MSC: Primary 42C05;

We consider the propagation of waves on a general non-homogeneous string. In terms of dynamics of waves we describe the Krein-Wiener logarithmic condition for the spectral density of a string:

$$\int_0^\infty \frac{\log v_{\mathrm{ac}}(\lambda)}{\sqrt{\lambda}(1+\lambda)}\, d\lambda > -\infty.$$

This condition plays a prominent role in the theory of stationary processes. We prove that it is equivalent to the existence of "asymptotically travelling" waves on the string with the given spectral measure. We also characterize Krein-Wiener strings in terms of their physical parameters (more precisely, in terms of mass distribution function). In particular, we show that strings made from two materials belong to the Krein-Wiener class if and only if the total mass of one of materials is finite. The talk is based on joint works with Sergei Denissov (University of Wisconsin–Madyson).

Approximation of branch points of an algebraic function from its holomorphic germ

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Let f be an algebraic function. How to construct a sequence of approximants to branch points of f if we know only the sequence of Taylor coefficients of some holomorphic germ of f? In particular, this problem has applications in molecular spectrocopy. The goal of the talk is to present the way to construct quickly (that is, exponentially fast) converging approximants to branch points with the help of Hermite–Padé polynomials of the first type constructed from the tuple of powers of the given germ of the function f.

Estimates Logarithmic Coefficient Inequalities for Certain Families of Analytic Functions

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2020 MSC: Primary 30C45.

In the process of proving the Bieberbach conjecture, Louis de Branges proved Milin's conjecture. The Milin conjecture states that for any analytic univalent function, the following inequality holds:

$$\sum_{m=1}^{n}\sum_{k=1}^{m}\left(k|\gamma_k|^k-\frac{1}{k}\right)\leq 0, \qquad n=1,2,\ldots,$$

where γ_k's are the coefficients in the series expansion of

$$\log\frac{f(z)}{z}=2\sum_{n=1}^{\infty}\gamma_n z^n.$$

These coefficients γ_n are called the logarithmic coefficients. Since then, works have been done on determining the bound for the logarithmic coefficients of functions in several subclasses. In this study, the the bound of logarithmic coefficients and other inequalities for a general family of starlike functions which are described by a subordination relation are established. Then, several special cases are deduced, which include one that corrects an earlier published result. This is joint work with Navneet Lal Sharma and Rosihan M. Ali.

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Possible orders of meromorphic solutions of linear difference equations with polynomial coefficients

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Keywords: Binomial series, linear difference equations, growth of order, polynomial coefficients, asymptotic solution, Poincaré-Perron theorem
2020 MSC: 39A45; 30D05.

We consider possible orders of transcendental meromorphic solutions of linear difference equations

$$P_m(z)\Delta^m f(z) + \cdots + P_1(z)\Delta f(z) + P_0(z)f(z) = 0, \tag{+}$$

where $P_j(z)$ are polynomials for $j = 0, \ldots, m$. Firstly, we give the condition on existence of transcendental entire solutions of order less than 1 of difference equations $(+)$. Secondly, we give a list of all possible orders which are less than 1 of transcendental entire solutions of difference equations $(+)$. Moreover, the maximum number of distinct orders which are less than 1 of transcendental entire solutions of difference equations $(+)$ are shown. Further, in both two cases, for a given difference equation $(+)$ with polynomial coefficients, we can construct a meromorphic solution of $(+)$ of order $\rho(f) = \rho$ for any $\rho \in [1, +\infty)$. Thirdly, for any given rational number $0 < \rho < 1$, we can construct a linear difference equation with polynomial coefficients which has a transcendental entire solution of order ρ. At least, some examples are illustrated for our main theorems. This is joint work with Katsuya Ishizaki, see [1] and [2].

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Non-linear resolvents of holomorphically accretive mappings

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Keywords: inverse function theorem, non-linear resolvent, distortion theorem, starlike mapping of order γ
2020 MSC: Primary 46G20; Secondary 47J07, 32H50.

In this talk I plan to discuss known and new results on holomorphically accretive mappings and their resolvents defined on the open unit ball of a complex Banach space. Namely, we will present a criterion for a mapping to be holomorphically accretive with given squeezing ratio of the generated semigroup as well estimates on its non-linear resolvents. Following an idea of Harris–Reich–Shoikhet, we establish an inverse function theorem for mappings that admit so-called one-sided estimates. This allows to obtain distortion and covering results for non-linear resolvents. In their turn, the distortion and covering theorems imply accretivity of resolvents with estimates on squeezing ratio. Furthermore, we prove that a nonlinear resolvent is a starlike mapping of given order subject some mild conditions.

Chord-arc domains, HQC mappings and beyond

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Keywords: harmonic quasiconformal maps, chord-arc curve
2020 MSC: Primary 30C62.

We will present generalizations of classical results of Astala, Zinsmeister and others to the case of harmonic quasiconformal mappings (HQC) obtained by Kalaj and pose some open problems. We will consider several ways to characterize geometry of domains in terms of BMO, Teichmuller spaces and welding maps.

The limiting set of iterations of entire functions on wandering domains

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Keywords: Limiting sets, Wandering Domains
2020 MSC: Primary 37F10; Secondary 30D05.

We will establish a continuum J whose boundary may arise as a limiting set of iterations of an entire function on an oscillating wandering domain and the interior may arise as an escaping wandering domain. If J has no interior, then J can be realized as a buried component of Julia sets. This discovery addresses a question posed by Osborne and Sixsmith. This is joint work with Jian-Hua Zheng.

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Modified Bohr phenomena for some classes of functions

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Keywords: Bohr Phenomena, Angular Derivative
2020 MSC: Primary 30B10, 30D45, 30H30; Secondary 30C80, 30A10.

In this talk, we consider two problems of recent interest.Firstly, modified Bohr radius for the class of analytic functions f such that $\operatorname{Re} f(z) > \alpha$ in the unit disk $|z| < 1$ and $\lim_{z\to 1} f(z) = \infty$ is investigated.Secondly, we determine the classical Bohr radius for the family of analytic α-Bloch functions f for which the Taylor expansion of f' has the form $f'(z) = \sum_{k=0}^{\infty} b_k z^k$. This is joint work with S.Ponnusamy and K-J.Wirths.

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Bohr-Rogosinski type inequalities for concave univalent functions

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Keywords: Bohr radius, Bohr-Rogosinski inequality, Concave Univalent functions, Subordination
2020 MSC: Primary 30A10, 30C45; Secondary 30C80.

This talk is based on Bohr-Rogosinski's inequalities and Bohr-Rogosinski property for the subfamilies of univalent functions defined on unit disk $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$ which maps to the concave domain, i.e., the domain whose complement is a convex set. All the results are proved to be sharp. This is joint work with Vasudevarao Allu.

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A generalization of the Bohr inequality and its applications

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Keywords: Bounded analytic functions, Bohr radius, Integral operators, Simply connected domains.
2020 MSC: Primary 30A10, 30H05, 40G05.

Bohr's classical theorem and its generalizations are now active areas of research and have been the source of investigations in numerous function spaces. In this talk, I present a generalized Bohr's inequality for the class of bounded analytic functions defined on the simply connected domain

$$\Omega_\gamma := \left\{ z \in \mathbb{C} : \left| z + \frac{\gamma}{1-\gamma} \right| < \frac{1}{1-\gamma} \right\}, \text{ for } 0 \leq \gamma < 1.$$

Part of its applications, we obtain the Bohr-type radii for some known integral operators.

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Fourier Transforms of Holonomic $\{D\}$-modules on the Complex Affine Line

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Keywords: Ordinary differential equations, Fourier transforms, Irregular singularity
2020 MSC: Primary 32C38; Secondary 34M35.

Let W_N be the Weyl algebra with N variables. An algebraic $\mathcal{D}$-module (i.e. a system of linear PDEs with polynomial coefficients) on the complex affine space $\mathbb{C}^N$ is naturally identified with a (left) W_N-module. Hence the Fourier transform for Weyl algebras induces Fourier transforms of holonomic $\mathcal{D}$-modules on the complex affine spaces. In the case of $N = 1$, we introduce irregular characteristic cycles and apply them to describe the exponential factors (i.e. the growth orders of the holomorphic solutions) of the Fourier transforms. We thus recover and moreover strengthen the classical stationary phase formula. This is a joint work with Kiyoshi Takeuchi.

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Robust signal recovery associated with the prolate spherical wave functions

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Keywords: Prolate spherical wave functions, signal recovery, Quaternion Prolate spherical wave functions
2020 MSC: Primary 33E10; 94A12; Secondary 11R52.

Signal recovery is a critical issue in the field of signal processing. This research presents a new strategy for signal recovery that uses prolate spherical wave functions (PSWFs), a type of function known for its effectiveness in this field. Historically, traditional signal recovery methods that use PSWFs have been based on the mean square error (MSE) metric, which assumes the presence of Gaussian noise. However, this reliance on MSE can be an issue when dealing with non-Gaussian noise, such as impulsive noise or outliers, because it is sensitive to these anomalies, which can lead to considerable errors in reconstruction. Our new approach differs from the norm by employing the maximum correntropy criterion (MCC), which is not affected by the nature of the noise distribution. This change enables our method to counteract the negative impacts of large, non-Gaussian noise components. Tests conducted on artificially generated signals with various types of noise have shown that our MCC-based signal recovery technique is more robust against a wider range of noise conditions compared to other current methods. The presentation also touches on further research into PSWFs in the context of Quaternions. This is joint work with Cuiming Zou.

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Countinuous non-extendability and holomorphic extendability of proper holomorphic mappings

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Keywords: proper holomorphic mappings, generalized pseudoellipsoids,
2020 MSC: Primary 32H35; Secondary 32H40.

X. Huang [1] proved that ; *Let $f : B^m \to B^n$ be a proper holomorphic mapping which extends as a C^2 mapping across the boundary. Assume that $m \leq n \leq 2m-2$. Under these assumptions, f is of the form $f(z) = (z, 0, \ldots, 0)$ up to the automorphisms of balls, where 0 is added $n-m$ components.* This theorem means that if the mapping is C^2 up to the boundary and the difference of dimensions is not so big, it is holomorphic extendable across the boundary. Under this line, the problem is whether the mapping is C^2 extendable or not. One answer to this problem is the following theorem, which is proved by F. Forstneric [4]; *For each integer $n \geq 1$, there is a proper holomorphic embedding $F : B^n \to B^N, N = n+1+2s$, where $s = s(n)$ is determined by n, such that F does not extend continuously to $\overline{B^n}$.* In this talk, we will show that the same phenomenon occurs for generalized pseudoellipsoids.

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Geometric Properties of Harmonic Shears on exterior unit disk

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2020 MSC: Primary 30C450.

Shear construction initiated by Clunie and Sheil-Small in 1984, is used to construct univalent harmonic mappings in unit disk by shearing a conformal mapping. However, on exterior unit disk, the shear technique does not guarantee the global univalence of constructed mappings, due to the limits at infinity could be different. In this talk, by using modified shear construction, we construct a class of bounded sense preserving univalent harmonic mappings, which maps the exterior unit disk onto finite bounded univalent components distributed in the complex plane, some accompanied graphics is given to illustrate these examples, and the boundary behavior are discussed.

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Range problems of shifted hypergeometric functions

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2020 MSC: Primary 30C45; Secondary 33C05.

In this talk, we study mapping properties of the shifted hypergeometric function $f(z) = z_2F_1(a, b; c; z)$ for real parameters with $0 < a \leq b \leq c$ and its variant $g(z) = z_2F_1(a, b; c; z^2)$. The orders of convexity of $f(z)$ are first given under certain conditions on the positive real parameters a, b and c. Then we show that the image domains of the unit disc under some shifted hypergeometric functions are convex and bounded by two lines. These results solve the range problems for f and g posed by Ponnusamy and Vuorinen in their 2001 paper. This is joint work with Toshiyuki Sugawa and Chengfa Wu.

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Univalence and quasiconformal extension criteria involving pre-Schwarzian and Schwarzian derivatives

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Keywords: Harmonic mapping, univalence and quasiconformal extension, pre-Schwarzian and Schwarzian derivatives, Loewner chain
2020 MSC: Primary 30C62; Secondary 30C55.

We wish to give some univalence and quasiconformal extension criteria, which involving pre-Schwarzian derivative and Schwarzian derivative. On one side, we use the method of Loewner chain to obtain Epstein type univalence conditions for locally univalent analytic functions. On the other hand, by using the pre-Schwarzian derivative of harmonic mappings, we obtain Ahlfors's type univalence and quasiconformal extension criteria for harmonic function. These results obtained extend the related results of earlier authors.

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Bohr's phenomena on a complex Banach space

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2020 MSC: Primary 32A05 ; Secondary 32A10.

Let $\mathbb{U}$ be the unit disc in $\mathbb{C}$, and let $f(z) = \sum_{n=0}^{\infty} a_n z^n$ be an analytic function from $\mathbb{U}$ to $\mathbb{U}$. Then, we have classical Bohr's inequality $\sum_{n=0}^{\infty} |a_n z^n| \leq 1$, for $|z| \leq 1/3$. It is called Bohr's phenomenon when an inequality of the above type holds in the disc $\{z : |z| < \rho_0\}$ with $0 < \rho_0 \leq 1$ for a class of analytic or harmonic functions. In this talk, we discuss about generalizations of several results related to Bohr's phenomena for locally univalent harmonic functions on $\mathbb{U}$ in $\mathbb{C}$ to pluriharmonic mappings on the unit ball of a complex Banach space. This is a joint work with Hidetaka Hamada.

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Schwarz's lemma and Beyond

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Keywords: Schwarz lemma, Lindelof's Inequality

Schwarz's lemma plays a significant role in geometric function theory and related areas. In this short survey talk, we explicate on various applications of Schwarz's lemma. We will discuss both versions of Schwarz- Pick lemmas. In particular, we expound on an inequality due to Lindelof. As a spin-off, we will discuss a $\delta - \epsilon$ form of Schwarz's lemma, which has applications in Teichmuller spaces.

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Study on the n-th Derivative of Bounded Analytic Functions

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Keywords: Schwarz-Pick lemma, Dieudonné's lemma, Hyperbolic derivative, Peschl's invariant derivative, Variability region
2020 MSC: Primary 30C80; Secondary 30F45.

Let $\mathcal{H}$ be the class of all analytic self-maps of the open unit disk $\mathbb{D}$. Denote by $H^n f(z)$ the n-th order hyperbolic derivative of $f \in \mathcal{H}$ at $z \in \mathbb{D}$. For $z_0 \in \mathbb{D}$ and $\gamma = (\gamma_0, \gamma_1, \ldots, \gamma_{n-1}) \in \mathbb{D}^n$, let $\mathcal{H}(\gamma) = \{f \in \mathcal{H} : f(z_0) = \gamma_0, H^1 f(z_0) = \gamma_1, \ldots, H^{n-1} f(z_0) = \gamma_{n-1}\}$. In this talk, we determine the variability region $V(z_0, \gamma) = \{f^{(n)}(z_0) : f \in \mathcal{H}(\gamma)\}$, which can be called "the generalized Schwarz-Pick Lemma of n-th derivative". We then apply the generalized Schwarz-Pick Lemma to establish a n-th order Dieudonné's Lemma, which provides an explicit description of the variability region $\{h^{(n)}(z_0) : h \in \mathcal{H}, h(0) = 0, h(z_0) = w_0, h'(z_0) = w_1, \ldots, h^{(n-1)}(z_0) = w_{n-1}\}$ for given $z_0, w_0, w_1, \ldots, w_{n-1}$. Moreover, we determine the form of all extremal functions.

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Boundary growth rates of superharmonic functions satisfying sublinear inequalities in the unit ball of $\{C\}^N$

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Keywords: boundary behavior, superharmonic function, Lane–Emden equation, Hausdorff dimension
2020 MSC: Primary 31B25; Secondary 31C05, 35J91.

Let $\Delta_{\mathbb{B}}$ be the Laplace–Beltrami operator associated with the Bergman metric on the unit ball $\mathbb{B}$ of $\mathbb{C}^N$ $(N \geq 2)$, let A be a measurable subset of $\mathbb{B}$ with positive Lebesgue measure, let $p \in (0, 1)$ and let $c_2 \geq c_1 > 0$. We estimate the anisotropic Hausdorff dimension of a set in $\partial\mathbb{B}$ where a positive superharmonic function satisfying

$$c_1 \chi_A u^p \leq -\Delta_{\mathbb{B}} u \leq c_2 u^p \quad \text{in } \mathbb{B}$$

grows faster than a prescribed order.

Norm estimates of higher order Hermite-Fej$\{\acute{\{e\}}\}$r interpolation for an Erdös-type weight

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2020 MSC: Primary 41A05; Secondary 41A25.

We discuss an interpolation polynomial for a weight w on $\mathbb{R}$. Let $\{p_n\}$ be the orthonormal polynomials with respect to w and let $\{x_{j,n}\}$ be the zeros of p_n. For an interger $\nu \geq 2$ and a continuous function f on $\mathbb{R}$, $H_n(\nu; f, \cdot)$ is defined by

$$H_n(\nu; f, x_{j,n}) = f(x_{j,n}) \text{ and } H_n^{(k)}(\nu; f, x_{j,n}) = 0$$

for $j = 1, 2, \cdots, n$ and $k = 1, 2, \cdots, \nu - 1$. It is called the ν-th order Hermite-Fejér interpolation polynomial for f based at $\{x_{j,n}\}$. Its degree is $n\nu - 1$. When $\nu = 1$ it is the Lagrange interpolation, $\nu = 2$ is the usual Hermite-Fejér interpolation and $H_n(4; f, \cdot)$ is called the Krylov-Stayermann interpolation polynomial. We assume $w \in \mathcal{F}(C^2+)$ and is regular (for definition see [1] and [5]) . We write $w(x) = \exp(-Q(x))$ and put $T(x) := xQ'(x)/Q(x)$. When T is unbounded, we say w is an Erdös-type weight. Set

$$\Phi_n(x) := \max\{1 - |x|/a_n, (nT(a_n))^{-2/3}\}$$

where a_n is the MRS number for w. The following new estimate is established.
Theorem. Let $w \in \mathcal{F}(C^2+)$ be regular and Erdös-type, and let $\nu \geq 2$ be even. Suppose that $wf' \in C_0(\mathbb{R})$ and $Q(x) \leq CQ'(x)$ $(x \geq 1)$. Then for any $0 < \eta < 1$ there exists a constant $C_\eta > 0$ such that

$$\|\Phi_n^{\nu/4} w^{\nu+1}(f - H_n(\nu; f, \cdot))\|_{L^\infty(\mathbb{R})} \leq C_\eta \frac{1}{n^{1-\eta}}$$

Note that when ν is odd, the above norm is unbounded in general. This is a joint work with Ryozi Sakai.

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Earthquake theorem for cluster algebras of finite type

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2020 MSC: Primary 51H10; Secondary 51M09.

Thurston defined the earthquake deformation and proved the earthquake theorem. The earthquake theorem states that for any two point of the universal Teichmüller space there is only one earthquake deformation. It was used to solve the Nielsen realization problem. Fomin and Zelevinsky defined the cluster algebra. Fock and Goncharov pointed out the relation between the cluster algebra and the Teichmüller space. In this talk, we introduce the earthquake deformation for the cluster algebra of finite type and prove the earthquake theorem for the cluster algebra of finite type. This is joint work with Tsukasa Ishibashi and Shunsuke Kano.

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Veech groups and tiling closures of origamis

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An origami (square-tiled surface), a covering $X \to E$ of the unit square torus E branched over one point $\infty \in E$, defines an arithmetic curve embedded in the moduli space. Möller showed that this embedding is arithmetic and that the Galois-Teichmüller theory on a particular origami yields another proof of the $\widehat{GT}$-relation of the absolute Galois group $G_{\mathbb{Q}}$. The Fuchsian group $\Gamma(X)$ of the embedded curve is a discrete $SL(2,\mathbb{Z})$-subgroup acting on the Teichmüller disk, called the Veech group. Schmithüsen characterized the Veech group of an origami as a stabilizer under a combinatorial group action on the free group $F_2 \cong \pi_1(E \setminus \{\infty\})$. In this talk, we present a tiling method to extract the 'covering part' of the membership criterion of Veech groups towards Bauer's example $Y \xrightarrow{36:1} X \xrightarrow{3:1} E$ for which $\Gamma(Y) = \Gamma(E) = SL(2,\mathbb{Z})$ and $\Gamma(X) \neq SL(2,\mathbb{Z})$.

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Normality criterion concerning total derivatives of holomorphic functions in $\{C\}^n$

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A family $\mathcal{F}$ of holomorphic mappings of a domain G in $\mathbb{C}^n$ into complex projective space $P^N(\mathbb{C})$ is said to be normal on G if any sequence in $\mathcal{F}$ contains a subsequence which converges uniformly on compact subsets of G to a holomorphic mapping of G into $P^N(\mathbb{C})$ and $\mathcal{F}$ is said to be normal at a point z_0 in G if $\mathcal{F}$ is normal on some neighborhood of z_0 in G. We continue investigating finding conditions involving values shared by holomorphic functions and their total derivatives which imply the normality for a family of holomorphic functions concerning the total derivatives in $\mathbb{C}^n$. We define $L_D^k(f) := \lambda_k D^k f + \lambda_{k-1} D^{k-1} f + \cdots + \lambda_1 Df + \lambda_0 f$, where $\lambda_k (\neq 0), \lambda_{k-1}, \ldots, \lambda_1, \lambda_0$ are complex constants. Consequently, we obtain the normality criterion of a family $\mathcal{F}$ of holomorphic functions f, where each function shares complex values with their linear total differential polynomials $L_D^k(f)$ in $\mathbb{C}^n$. This is joint work with Molla Basir Ahamed.

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On Closedness of Subvarieties of Bands

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In this paper, we have proved that all subvarieties of the variety of left (right) regular bands are closed in the variety of n-nilpotent extension of bands. Also, we have tried to explore the closedness of rectangular bands in the variety $V = [ac = ab^n c](n \in N)$ of semigroups. Furthermore, we have demonstrated that all subvarieties of the variety of left (right) normal bands are closed in the variety $V = [axy = a^p y^q x^r](p, q, r \in N)$ of semigroups and the closedness of all subvarieties of the variety of left (right) quasinormal bands in the variety $V = [axy = a^p x^q a^r y](p, q, r \in N)$ of semigroups is also discussed in this paper. This is joint work with Shabnam Abbas and Wajih Ashraf.

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Nilpotent Bicomplex Matrices and Operators Theoretical Insights and Relationships

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This work investigates different kinds of nilpotent bicomplex linear operators and matrices. It delves into the connection between them and presents various results. It also includes examples and counterexamples to substantiate these results.

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Some results on the eigenvalues of bicomplex operator $T = e\_1T\_1 + e\_2T\_2$

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This paper discusses the properties of linear maps on bicomplex numbers. We define the concept of eigenvalues and eigenvectors of a particular type of bicomplex operators $T = e_1T_1 + e_2T_2$ and discuss some related properties of such eigenvalues and eigenvectors.

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On the centers of Blaschke polygons

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Let $E_2 \subset E_1$ be a pair of nested ellipses and P an n-sided polygon inscribed in E_1 and circumscribed about E_2. Poncelet's theorem guarantees that for any point of E_1, there exists an n-sided polygon inscribed in E_1 and circumscribed about E_2, which has this point as one of its vertices. Such an n-sided polygon is called an *n-sided Poncelet polygon*. For each n-sided Poncelet polygon P we consider the following "centers", the mean center $C_v(P)$ and the centroid $C_l(P)$. The following Theorem is an interpretation of Shestakov's Theorem (1814).

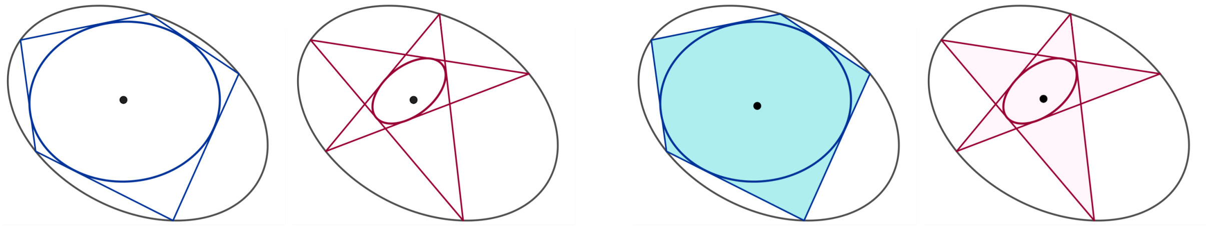

Figure 1: The left figures indicate the mean centers of a Poncelet pentagon and pentagram as black dots. The black dots in the right figures are centroids.

Shestakov's Theorem ([ST16]) Let $E_2 \subset E_1$ be a pair of nested ellipses that admit a 1-parameter family of Poncelet n-sided polygons P_t. Then both loci $C_v(P_t)$ and $C_l(P_t)$ are ellipses similar to E_1 or single points. In this talk, we consider the locus of the "center" for the polygon constructed by the preimages of Blaschke products. Let B be a Blaschke product of degree d,

$$B(z) = e^{i\theta} \frac{z - a_1}{1 - \overline{a_1} z} \cdots \frac{z - a_d}{1 - \overline{a_d} z}, \quad (a_k \in \mathbb{D},\ \theta \in \mathbb{R}).$$

In the case that $\theta = 0$ and $B(0) = 0$, B is called *canonical*. Let $z_1, \cdots, z_d$ be the d distinct preimage of $\lambda \in \partial\mathbb{D}$ by B and l_λ the set of lines joining z_j and z_k with $j \neq k$. Then, the envelope I_B of the family of lines $\{l_\lambda\}_{\lambda \in \partial\mathbb{D}}$ is called the *interior curve associated with* B. Moreover, a polygon with vertices at $z_1, \cdots, z_d$ is called a *Blaschke polygon*. For $d = 3$, the interior curve associated with B is an ellipse [DGM02], and each Blaschke triangle with vertices $B^{-1}(\lambda)$ ($\lambda \in \partial\mathbb{D}$) is exactly the Poncelet triangle [Fra04]. For $d > 3$, the interior curve is not always an ellipse (cf. [Fuj13]). Even so, analogous to Shestakov's theorem, we obtain the following result. **Theorem 1** Let B be a canonical Blaschke product of degree d and $z_1, \cdots, z_d$ the d distinct preimages of $\lambda \in \partial\mathbb{D}$ by B. As λ ranges over $\partial\mathbb{D}$, the mean center $w = (z_1 + \cdots + z_d)/d$ of d-sided polygon with vertices $z_1, \cdots, z_d$, forms a circle or a single point. In addition, we discuss results that are extensions of Theorem 1 by introducing Blaschke-like maps (see [FG23] for details) defined on the domain whose boundary is an ellipse or a parabola.

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Distortion, Radius of Concavity and Several Other Radii Results for Certain Classes of Functions

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We consider the class of all meromorphic univalent functions in the unit disc $\mathbb{D}$ with a simple pole at $z = p$ and normalized by the conditions $f(0) = 0$ and $f'(0) = 1$ and denote this class of functions by $S(p)$. We derive the region of variability of the quantity $zf''(z)/f'(z)$ and establish an estimate of the quantity $|zf'(z)/f(z)|$ for $f \in S(p)$. We define radius of concavity and compute the same for $S(p)$ and some other well-known classes of functions. We also explore the linear combinations of functions belonging to some well-known classes and investigate their radii of univalence, convexity and concavity. This is joint work with Bappaditya Bhowmik.

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On meromorphic harmonic functions with a pole at the origin

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In this talk, we investigate meromorphic univalent harmonic functions having a simple pole at the origin. Our investigation begins with establishing sufficient conditions that ensure the univalence of these functions within the larger class of meromorphic harmonic functions. We then delve into coefficient estimates for certain geometric subclasses of these meromorphic univalent harmonic functions. Subsequently, we provide several necessary and sufficient conditions for f to be hereditarily λ-spirallike. Finally, we offer a comprehensive characterization of hereditarily meromorphic harmonic Archimedean and hyperbolic spirallike functions. This is joint work with A. Sairam Kaliraj.

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Bounds for the Derivative of Certain Meromorphic Functions and on Meromorphic Bloch-type Functions

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It is known that if f is holomorphic in the open unit disc $\mathbb{D}$ of the complex plane and if, for some $c > 0$, $|f(z)| \leq 1/(1-|z|^2)^c$, $z \in \mathbb{D}$, then $|f'(z)| \leq 2(c+1)/(1-|z|^2)^{c+1}$. We consider a meromorphic analogue of this result. Furthermore, we introduce and study the class of meromorphic Bloch-type functions that possess a nonzero simple pole in $\mathbb{D}$. In particular, we obtain bounds for the modulus of the Taylor coefficients of functions in this class. This is joint work with Bappaditya Bhowmik.

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Univalence of horizontal shear of Ces´aro type transforms

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2020 MSC: Primary 30C45, 31A05, 31A10; Secondary 30C55, 30E20,

The symbol $\mathcal{A}$ denotes the class of all analytic functions φ in the unit disk $|z| < 1$ with the normalization $\varphi(0) = 0$ and $\varphi'(0) = 1$. Some of the subclasses of $\mathcal{A}$ containing univalent functions are the class of starlike and convex functions. Alexander transform plays an important role in geometric function theory as it gives a relationship between the starlike and convex functions. Univalence properties of the Alexander as well as the Cesáro transforms are classical problems already studied in literature. In this presentation, for an analytic function $\varphi \in \mathcal{A}$, we consider a generalized complex integral transform which incorporate both Alexander and Cesáro transforms defined by

$$C_{\alpha\beta}[\varphi](z) = \int_0^z \left(\frac{\varphi(z)}{z(1-z)^\beta}\right)^\alpha d\zeta$$

In particular, $C_{10}[\varphi]$ and $C_{11}[\varphi]$ respectively represent the Alexander and the Cesáro transforms. Our main focus is to find the condition on the parameters $\alpha, \beta \in \mathbb{C}$ for which the integral transform $C_{\alpha\beta}$ and its harmonic analog through horizontal shearing are univalent whenever φ is a normalized univalent function. As applications to our main results, a few non-trivial univalent harmonic mappings generated by the method of shear construction are also presented. This is a joint work with Prof. S. K. Sahoo.

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Geometric structures on the Heisenberg group

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2020 MSC: Primary 32V15; Secondary 53C17.

Geometric structures on the Heisenberg group serve as powerful tools in complex hyperbolic geometry. These structures provide insights into the boundary behavior of hyperbolic spaces, facilitate the study of isometries and automorphisms, and contribute to the understanding of rigidity and deformation problems. In this talk, we will focus primarily on CR structures and Sasakian structures on the Heisenberg group. Additionally, we will demonstrate how, using the CR structure of the Heisenberg group, we can construct several different Kähler structures in the Siegel domain. This is a joint work with Ioannis D. Platis and Joonhyung Kim.

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On Yau sequence over complete intersection surface singularities of Brieskorn type

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In this paper, we study the Yau sequence concerning the minimal cycle over complete intersection surface singularities of Brieskorn type, and consider the relations between the minimal cycle and the fundamental cycle. Further, we also give the coincidence between the canonical cycles and the fundamental cycles from the Yau sequence concerning the minimal cycle.

The Bergman kernels of Roos type domains explicit forms and zeros

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Although the Bergman kernel has a long history of research after its discovery by Stefan Bergman, it is a hard problem to find a complex domain $D \subset \mathbb{C}^n$ whose Bergman kernel K_D has an explicit expression. In this talk, we give new examples of explicit Bergman kernels for certain Roos type domains whose definition is inspired by Roos' paper [1]. Moreover we also study the Lu Qi-Keng problem for Roos type domains which asks the Bergman kernel is zero-free.

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On the Azukawa pseudometric defined from the pluricomplex Green function with poles along subvarieties

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The Azukawa pseudometric (cf. [Azu1]) is a function defined from the pluricomplex Green function with a pole at a point, and it is a generalization of the Robin constant defined from the classical Green function. The Azukawa pseudometric is useful to analyze behavior of the pluricomplex Green function near its a pole, and it has deep connections with important objects in complex analysis. In this talk, we consider the counterpart of the Azukawa pseudometric for the pluricomplex Green function with poles along subvarieties (cf. [Kik]), and explain about its properties and applications.

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Skoda-Zeriahi type integrability and entropy compactness for Poincarŕtype Kḧler metrics

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2020 MSC: Primary 31U05.

Zeriahi[3] proved a uniform version of Skoda's integrability theorem of plurisubharmonic functions. This result has an important application to the study of constant scalar curvature Kähler metrics on compact Kähler manifolds. In this talk, we consider the integrability theorem for the singular setting. More precisely, we consider the Skoda-Zeriahi's integrability theorem for the singular measure defined by a Poincaré type Kähler metric. As an application, we talk about some compactness result of the relative entropy on the finite energy space (see [1]). This work is motivated by the variational characterization of constant scalar curvature metrics by Chen-Cheng[2].

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almost SC^*-normal spaces

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In this paper,we introduced the concept of Normal spaces called almost SC*- normal spaces by using SC*-open set and obtained several properties moreover we obtained some new characterization and preservation theorem of almost SC*- normal Space.

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Some results on Bicomplex numbers

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This paper explores essential new fundamentals concerning bicomplex numbers, discusses the algebraic behavior of bicomplex numbers with respect to their three types of ordinary conjugates, and explains the difference between idempotent components of such conjugates and the conjugates of these Idempotent components.

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Some results on Constacyclic codes and Quantum codes over Finite Chain Ring and Finite Field

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Constacyclic codes are a well-known class of linear codes that contain many optimal codes and possess excellent error-correcting properties. These attributes make constacyclic codes highly effective for encoding and decoding using linear shift registers, which is why they are favored in engineering applications. Constacyclic codes generalize cyclic and negacyclic codes. With the discovery that nonlinear codes relate to linear codes over the ring of integers modulo four, the algebraic structure of constacyclic codes over finite rings has become an intriguing problem. In this context, I first determine the algebraic structure of all ∂-constacyclic codes of length $4p^r$ and their duals over the finite commutative chain ring $F_{p^t} + uF_{p^t}$, where ∂ is an arbitrary unit in $F_{p^t} + uF_{p^t}$, p is an odd prime, and l and r are positive integers. Quantum MDS (Maximum Distance Separable) and AMDS (Almost MDS) codes are crucial in quantum communication systems for protecting the transmission of quantum information over long distances. In my work, I construct non-binary quantum codes using repeated root cyclic and negacyclic codes of length $4p^r$ over F_{p^t}. Additionally, I investigate the structure of all MDS and AMDS cyclic and negacyclic codes of length $4p^r$ over F_{p^t}. Through CSS constructions, I also obtain numerous new quantum MDS and AMDS codes.

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On a question of Gary G. Gundersen concerning meromorphic functions sharing three distinct values IM and a fourth value CM

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In 1992, Gundersen [1] proposed the following famous open question: if two non-constant meromorphic functions share three values IM and share a fourth value CM, then do the functions necessarily share all four values CM? The open question is a long-standing question in the studies of the Nevanlinna′s value distribution theory of meromorphic functions, and has not been completely resolved by now. In this paper, we prove the following result: suppose that f and g are two distinct non-constant meromorphic functions, and one of f and g has finite order. If f and g share a_1, a_2, a_3 IM and a_4 CM, where a_1, a_2, a_3, a_4 are four distinct complex values in the extended complex plane, then f and g share a_1, a_2, a_3 and a_4 CM. Applying the main result obtained in this paper, we completely resolve a question proposed by Gundersen in [2, p.458] concerning the nonexistence of two distinct non-constant meromorphic functions sharing three distinct values DM and a fourth value CM. The obtained result also improves the corresponding result in Mues[3, pp.109-117] concerning the nonexistence of two distinct non-constant entire functions that share three distinct finite values DM. Examples are provided to show that the main results obtained in this paper, in a sense, are best possible.

This is joint work with Qing-Fei Zhai and Hong-Xun Yi.

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