Quantitative homogenization in nonlinear elasticity

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We consider a nonlinear elastic composite with a periodic microstructure described by the nonconvex energy functional

$$\int_\Omega W(\tfrac{x}{\varepsilon},\nabla u(x)) - f(x)\cdot u(x)\,dx.$$

It is well-known that under suitable growth conditions the energy Γ-converges to a homogenized functional a homogenized energy density W_{hom}. One of the main problems in homogenization of nonlinear elasticity is that long-wavelength buckling prevents the possibility of homogenization by averaging over a single period cell, and thus W_{hom} is in general given by an infinite-cell formula. Under appropriate assumptions on W (frame indifference, minimality at identity, non-degeneracy) and on the microstructure (e.g., possibly touching smooth inclusions), we show that in a neighbourhood of rotations W_{hom} is characterized by a single-cell homogenization formula. In particular, we prove that correctors are available—a property that we exploit to derive a quantitative two-scale expansion and uniform Lipschitz estimates for minimizers. This is joint work with Mathias Schäffner (TU Dresden).